IDENTIFICATION OF MATHEMATICAL DESCRIPTION OF THE DYNAMICS OF EXTRACTION OF OIL MATERIALS IN THE ELECTRIC FIELD OF HIGH FREQUENCY

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ABSTRACT. One of the most important stages of the technological process of extraction of target components from oilseeds is extraction. This stage lasts the longest, and therefore, it generally determines the speed and cost-effectiveness of the whole process. A promising direction for effective organization of the extraction process is involving microwave technologies, the use of ultra-high frequency electromagnetic field (EHF) microwave energy directly in the technological process.

The complex nature of the interaction of the factors that determine the intensity of the extraction process in the microwave field does not allow establishing (create) an exact mathematical model of extraction. We propose a method of parametric identification of the mathematical description of the extraction dynamics. This method allows determining the kinetic coefficients of the process from one experimental experiment on the existing installation. A simplified process mechanism was chosen for the study and a general description of the phenomena of heat and mass transfer in a capillary-porous body was used for the known general description. The parameters of the obtained model were identified according to experimental studies. The thus obtained model of non-stationary processes can be used to optimize the parameters and automate the technical means of extraction of oil materials.

Introduction

Analysis of recent research and publications

The main provisions of the theoretical foundations of extraction in the system "solid-liquid" are covered in fundamental works (Romankov, Frolov, 1990; Akselrud, Lysianskij, 1974; Beloborodov, 1999; Perez, et al., 2011), including the use (microwave EMF) of the microwave field (Burdo, 2005; Burdo et al., 2015; Sánchez et al., 2019). The theoretical substantiation of the efficiency of the use of the electromagnetic field of ultra-high frequency for the intensification of the intravolumetric processes of heat and mass transfer, including in the process of extraction of vegetable and oil materials is given in (Burdo, 2005; Sánchez et al., 2019; Burdo et al., 2017). An analysis of approaches to the mathematical description of the processes of heat and mass transfer in a solid body and under the action of a microwave field is presented in (Sánchez et al., 2019; Kotov, Bandura, 2018). Experimental studies of the extraction process of vegetable oil-containing materials confirmed the intensifying effect of the microwave field on the removal of the target component (Sánchez, 2017; Bandura, 2018), which is a significant increase in oil yield. It is established that the physical basis for increasing the yield of the target component is the barofusion component of the process: a sharp increase in the intensity of vaporization and, accordingly, the pressure of the vapour-liquid mixture, which leads to the ejection of the droplet liquid phase from capillaries (microcapillaries) and pores. Thus, it can be assumed that the mechanism of extraction of the target component (oil matter) from the capillary-porous body under the action of the microwave field consists of mass diffusion (which takes into account all types of transfer from capillaries, microcapillaries and pores) and barothermal component (heating, vapor, heat, and steam) and the removal of the vapor-liquid phase from...
the capillaries, while the vapor phase condenses on the colder walls of the solid skeleton. Still, analysis of existing model representations showed that even with very simple simplifications, the description in physical phenomena, analytical dependencies that are suitable for practical calculations can only be approximated by experimentally obtained data. So, the most appropriate and promising way of obtaining a mathematical model of the extraction process will be to use the inverse problem method (Alifanov et al., 2009) on the basis of experimental information about an object realizable by the Levenberg-McVard algorithm (Burdo, 2010).

**Formulation of the problem.** Extraction in the dispersed system of vegetable oil materials is widespread in oil production technologies. A promising direction for effective organization of the extraction process is involving microwave technologies, the use of ultra-high frequency electromagnetic field (EHF) microwave energy directly in the technological process.

Due to the imposition of microwave EMF on the interacting phases during the extraction of oil raw materials, it is possible to obtain a concentrated extract, shorten the duration of the technological process and significantly identify it while reducing the specific energy consumption. To determine the rational modes of the process and the parameters of the facilities that implement it, it is necessary to have a mathematical description (mathematical model) that adequately describes the process under study. The complex nature of the interaction of the factors that determine the intensity of the extraction process in the microwave field does not allow establishing (creating) an exact mathematical model of extraction. Therefore, for the study, it is possible to choose a simplified mechanism of the process and to use for its formalization a well-known general description of the phenomena of heat and mass transfer in a capillary-porous body, and to identify the parameters of the obtained model according to experimental studies. The model of non-stationary processes thus obtained can be used to optimize the parameters and automate the technical means of extracting oil materials.

The purpose of the work is to improve the mathematical model of the process of extraction of oil raw materials in the electromagnetic field of ultra-high frequency to intensify its parameters according to experimental studies.

**Material and methods**

Experimental studies were performed to determine the parameters of microwave processing, which is the highest yield of the target components. Studies on the kinetics of extraction were performed with the rapeseed variety "Ozymyi" (bagasse and whole grain), determining the effect on the process of the following process parameters: the magnitude of the power (N, W) of the pulsed electromagnetic field (IEM field), hydro module extract (the ratio of the mass of the solution to the dry weight of the raw material), temperature (t, °C), extraction time (τ, s.). The solvents used were C\textsubscript{2}H\textsubscript{5}OH alcoholic hexane C\textsubscript{6}H\textsubscript{14}. During the experimental studies control devices, equipment and experimental microwave stands, including the development of the authors, were used (Alifanov et al., 2009). In a laboratory installation, the power of a microwave amplifier can vary from 0.4 to 1.6 kW. The microwave frequency is 2450 MHz. To study the kinetics of the extraction process, the concentration of the solution was determined. Extraction of the oil from the micelle to determine the concentrations was performed arbitrarily by evaporation of the solvent. To obtain a comparative study of the kinetics of oil extraction from rapeseed, the study was performed in two stages, using the usual method (in the thermostat TS-80) and using microwave radiation. In the usual way, a container with crushed rapeseed bagasse and solvents was placed in a thermostat. The thermostat maintained the temperature inside the vessels at the same level during the extraction process. To study the oil concentration, samples were taken every 10 minutes. They were weighed and placed in a desiccator SPT-200, where the solvent was evaporated from the test sample, and then weighed on an analytical balance VLA-200G-M.

The main elements of the experimental microwave stand (Fig. 1) were the cabinet, as a result of which a microwave field was created and a vessel in which the extraction process itself continued. The rack offered microwave power control. The principle of operation of the experimental stand is as follows: in the vessel with the product 3, the process of extraction was occurring under the influence of a microwave field in the chamber 1.

The extractant vapour enters the reflux condenser 2, condenses and flows back into the reaction vessel together with the test sample and solvent. Using a syringe, five micelles were collected for further study. To ensure the reliability of the results, the experiments were performed in six-fold replication, and in case of significant scattering of results (with a coefficient of variation of more than 15%), the number of repetitions increased to 10. Analytical calculations were performed using software packages for PC: MathCAD 14 M010, Excel 11.0 (Office 2004).
Results

The total specific mass content of the extractable substance was equal to the sum of the specific mass content of the substance in the liquid phase and as a vapour:

\[ u = u_p + u_n, \]

where \( u \) – mass concentrations of the extractable substance in the solid (for the liquid phase \( u_p \) and a vapour \( u_n \), kg kg\(^{-1}\)).

As the mass of the extractable substance may have varied due to phase transformation \( du \) and by transferring the substance by diffusion \( du_d \) and due to the transfer of the substance in the droplet in the vapour pressure phase \( du_n \), the following equality holds:

\[ du = du_f + du_d + du_n \]

We introduced according to (Burdo et al., 2010) criteria for the phase transformation of the extractable substance \( \varepsilon, \varepsilon' \), according to the ratios:

\[ \varepsilon = \frac{du_n}{du}, \quad \varepsilon' = \frac{du_p-n}{du} \]

where:

\( \varepsilon \) – determines the ratio of the substance, which was released in the form of steam, to all the removed substance;

\( \varepsilon' \) – determines the ratio of the dropping substance, which was removed under the pressure of the formed steam (Lykov, 1971; Burdo, 2010).

There was a relationship between the values \( \varepsilon \) and \( \varepsilon' \): \( \varepsilon' = (1-\chi)\varepsilon' \) (\( \chi \) – the criterion of thermomechanical extrusion of the liquid phase).

Then the local rate of change of the mass content of the extractable substance would have been:

\[ \frac{du}{dt} = \varepsilon \frac{du}{dt} + \varepsilon' \frac{du}{dt} + \frac{du_d}{dt}. \]

Based on the above relations we obtained the differential equation of mass transfer of the extractable substance:

\[ \rho_0 \frac{du}{dt} = a_m \nabla^2 u + \varepsilon \frac{du}{dt} + \varepsilon' \frac{du}{dt} + \frac{du_d}{dt}. \]

where:

\( \rho_0 \) – the density of solids, kg m\(^{-3}\);

\( a_m \) – the coefficient of mass conduction, m\(^2\) s\(^{-1}\).

In equation (5), the first component of the right part determined the diffusion component of the transfer of the extracted substance, the second was the source of the vapour phase flow and the third was the source of the droplet-liquid flow.

Having transferred the last component to the left part of the Eq. 5 and having replaced the mass concentration with volume \( (\rho_0 dt = c) \) we rewrote equation (5) in the form:

\[ \frac{dc}{dt} = D_{ef} \nabla^2 c + \varepsilon \rho_0 \frac{du}{dt}. \]

where \( c \) – volumetric concentration of the extractable substance in the solid, kg m\(^{-3}\); \( D_{ef} = D(1-\varepsilon') \) – effective diffusion coefficient takes into account the transfer of the liquid phase of substances.

Given that the component \( \varepsilon \rho_0 \frac{du}{dt} = q_m \) was the source of the flow of matter in the form of steam, Eq. 6 can be written as follows:

\[ \frac{dc}{dt} = D_{ef} \nabla^2 c + q_m. \]

The obtained equation determined the dynamics of the extraction process in the presence of an internal source of matter (in the form of steam).

The equation of transfer in a solid capillary-porous body filled with a substance that was extracted in the presence of an internal energy source (in this case, the equation of thermal conductivity of (Eq. 16):

\[ \frac{\partial \theta}{\partial t} = a_m \nabla^2 \theta + \varepsilon \frac{1}{c_m} \frac{\partial u}{\partial t} + \frac{\rho_0}{c_m \rho_0}, \]

where:
\[ \theta - \text{the temperature of the solid, } ^\circ\text{C}; \]
\[ r - \text{specific heat of vaporization, J kg}^{-1}; \]
\[ c_m - \text{specific heat of the material, J (kg}^{-1}\text{)C}^{-1}; \]
\[ P_r - \text{capacity of volumetric heat dissipation, W m}^{-3}. \]

By replacing \( \frac{du}{dt} \) with the obvious equation that followed from the definition of the Kosovich criterion \((Ko \cdot \frac{r}{c_m} - \frac{du}{dt} = cm \cdot Ko \cdot \frac{a}{ct} \cdot \theta)\) we wrote Eq. 8 in the form:

\[ \frac{\partial \theta}{\partial t} = a_c \nabla^2 \theta - \frac{q_v}{c_p0}. \quad (9) \]

where:
\[ a_e = a_m(1 - Ko); \]
\[ q_v = \frac{P_v}{c_m r_0}. \]

The differential diffusion Eq. 7 in the form:
\[ \frac{\partial c}{\partial t} = D_\text{ef} \nabla^2 c + \varepsilon \frac{c_m}{r} Ko \frac{\partial \theta}{\partial t}. \quad (10) \]

The value \( \varepsilon Ko \frac{\partial \theta}{\partial t} \) had dimension (kg (m³ s)⁻¹) and characterizes the intra-volumetric steam mass flow initiated by the rate of change of temperature.

To identify the parameters of mass transferred during the extraction process under the action of the microwave field, according to the algorithm (17, 18) it was desirable to have one dependence in the form of the mass conduction equation.

Quantity \( q_m = \varepsilon r_0 \frac{\partial \theta}{\partial t} \) determined the amount of extractable substance vapour emitted per unit time (i.e. the rate of pair formation).

The rate of vaporization could have been determined from the equation of thermal balance for the particle mass \( m_0 \), which was heated “in contact” with the extractant electromagnetic field of power \( P \). The heat allocated in the volume of the particles of plant material was spent on heating the capillary-porous body, evaporation of the liquid phase and was transmitted by convection to the volume of the extractant:

\[ P_m = m_m c_m \frac{\partial \theta}{\partial t} + \varepsilon m_0 r \frac{du}{dt} + \alpha f(\theta - \theta_p). \quad (11) \]

where:
\[ m_m - \text{the mass of solid fraction, kg;} \]
\[ \alpha, f - \text{coefficient and surface of heat exchange between phases, W (m}^{-2}\text{)C}^{-1} \text{ and m}^{-1}; \]
\[ c_m - \text{specific heat of solid phase, J (kg}^{-1}\text{)C}^{-1}; \]
\[ m_0 - \text{mass of material and solution, kg;} \]
\[ \theta_p - \text{solvent temperature, } ^\circ\text{C}. \]

The following equation was written for the volume of the extractant:

\[ \frac{\partial c(x,t)}{\partial t} = D_\text{ef} \frac{\partial^2 c(x,t)}{\partial x^2} + q_m, \quad \frac{\partial c(x,t)}{\partial t} = D_\text{ef} \frac{\partial^2 c(x,t)}{\partial x^2} + q_m, \]

where \( \beta - \text{coefficient of mass transfer, m s}^{-1}. \)

The solution of system (17) if \( q_m = \text{const} \) by analogy with (Eq. 16) was given by:

\[ \frac{c(x, \tau) - c_p}{c_{o-p}} = \frac{P_o'}{2} \left( 1 - \frac{x^2}{R^2} + \frac{2}{B_i m} \right) - \sum_{n=1}^{\infty} \left( 1 + \frac{P_o'}{\mu_n^2} \right) A_n \cos \mu_n \frac{x}{R} e^{-\mu_n^2 \tau}, \quad (18) \]

where:
\[ V_p - \text{volume liquid phase, m}^3; \]
\[ c_p - \text{density of the liquid phase, kg m}^{-3}; \]
\[ c_p - \text{specific heat of liquid phase, J (kg}^{-1}\text{)C}^{-1}. \]

Adding Eqs. 11 and 12 we have:

\[ P_m + P_p = m_m c_m \frac{du}{dt} - m_0 r \frac{du}{dt} + m_p c_p \frac{\partial \theta}{\partial x} = N \eta, \quad (13) \]

where:
\[ N - \text{the power source of an electromagnetic emitter, W;} \]
\[ P_m - \text{heat release in the extractable of solid fraction, W;} \]
\[ P_p - \text{heat release in the extractable of the liquid phase, W;} \]
\[ m_m, m_p - \text{mass of solid fraction and liquid, kg;} \]
\[ \eta - \text{the efficiency of the installation of electromagnetic radiation.} \]

In Eq. 13, the component \( m_p c_p \frac{\partial \theta}{\partial x} = P_p' \) was numerically equal to the power of heat dissipation in the volume (mass) of the extractant when neglecting the heat loss to the environment.

Using the replacement of the rate of heating of the body (derivative temperature in time) by the obvious ratio:

\[ \frac{\partial \theta}{\partial t} = \frac{r}{c_m} R_b \frac{du}{dt}, \quad (14) \]

after the transformations we got:

\[ \frac{du}{dt} = \frac{m_0 r}{(m_n r_0 + m_0 r)} R_b + m_0 r \frac{du}{dt}, \quad (15) \]

where \( R_b = \frac{c_m}{r} \frac{du}{dt} \) - Rebindeer criteria.

Substituting the value (15) into Eq. 6 we had:

\[ \frac{\partial c}{\partial t} = D_\text{ef} \nabla^2 c + \frac{(N_x - P_p') \varepsilon m_0}{(m_n r_0 + m_0 r)} R_b. \quad (16) \]

Thus obtained differential equation of diffusion of the extractable substance in the capillary cell under the action of the microwave field:

\[ \frac{\partial c}{\partial t} = D_\text{ef} \nabla^2 c + q_m. \quad (16a) \]

The structure of Eq. 16a was analogous to the equation of thermal conductivity of a body with an internal heat source whose solution is known (Eq. 19).

Setting the initial and boundary conditions of the third kind for the body in the form of an unlimited plate length 2R writing as (if \( \tau = 0; \quad \frac{\partial c}{\partial x} = 0; c = c_o; x = R; \quad -R < x < +R \) equation):

\[ \frac{\partial c}{\partial x} = D_\text{ef} \nabla^2 c + q_m. \quad (17) \]
where:

\( c_0 \) – initial concentration, kg m\(^{-1}\);  
\( c_p \) – concentration of the extractable substance in the solvent (extractant), kg m\(^{-1}\);  
\( P\omega' = \frac{q_m R^2}{D_{ef}(c_p-c_0)} \) – Pomerantsev criterion, mass transfer;  
\( B_{im} = \frac{BR}{D_{ef}} \) – criterion of Biomass transfer;  
\( \mu_n = B_{im} \eta \mu_n \) – roots of the characteristic equation;  
\( A_n \) – coefficient;  
\( F_{om} = \frac{D_{ef}}{R^2} \) – Fourier criterion.

For the average concentration of the extractable substance:

\[
\bar{c}(\tau) = \frac{1}{R} \int_{0}^{R} c(x, \tau) \, dx, 
\]  
(19)

kinetic dependence was determined by the equation:

\[
\frac{\bar{c}(\tau) - c_p}{c_0 - c_p} = \frac{P\omega'}{3} \left( 1 + \sum_{n=1}^{\infty} \left( 1 + \frac{P\omega'}{\mu_n^2} \right) B_n e^{-\mu_n^2 \omega' \tau} \right), 
\]  
(20)

Eqs 18 and 20 indicate:

\[
A_n = \frac{2 \sin \mu_n}{\mu_n + \sin \mu_n \cos \mu_n}, B_n = A_n \frac{\sin \mu_n}{\mu_n} \]  
(21)

To determine the change in the average value of the concentration of the extractable substance from the "elementary" particle according to the experiments (extraction kinetics) we used the equation of material balance:

\[
V_m(c_0 - \bar{c}) = V_p(c_p - c_{p0}), 
\]  
(22)

where:

\( c_{p0} \) – initial concentration of the extractable substance, kg m\(^{-1}\);  
\( V_m = \frac{m_m}{\rho_m} \) – volume of the solid phase, m\(^3\);  
\( V_p = \frac{m_p}{\rho_p} \) – volume of the liquid phase, m\(^3\).

Using the notion of a hydro module \( n = \frac{V_m}{V_p} = \frac{m_m \rho_p}{\rho_m \rho_p} \) and considering that the initial concentration of the extractable substance \( (c_{p0}) \) was zero from Eq. 22 we obtained:

\[
\bar{c}(\tau) = \bar{c}_0 - \frac{1}{n} c_p(\tau), 
\]  
(23)

where \( c_p(\tau) \) was a change in time of concentration of the substance in the extractant, kg m\(^{-1}\).

According to the results of experimental studies, kinetic dependences of the concentration change in the extractant were obtained (Fig. 2).
To reduce the influence of random and systematic errors that occur during the experiments on the result of identification (determination of the coefficients of equation (17), (20), the experimental data were approximated by the dependence in the form:

$$\bar{c}_p(\tau) = c_{p,0}(1 - e^{-k\tau}),$$

(24)

where:

- $c_{p,0}$ was the value of the set (equilibrium) concentration value, kg m$^{-3}$;
- $k$ – extraction coefficient on the extractant, 1 s$^{-1}$.

The parameters of the experimental kinetics were shown in Table 1.

<table>
<thead>
<tr>
<th>N, W</th>
<th>$c_{p,0}$, kg m$^{-3}$</th>
<th>$k$, 1 s$^{-1}$</th>
<th>$t_{25}$, °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>127</td>
<td>66.0</td>
<td>0.080</td>
<td>78.3</td>
</tr>
<tr>
<td>225</td>
<td>73.0</td>
<td>0.082</td>
<td>78.3</td>
</tr>
<tr>
<td>425</td>
<td>78.3</td>
<td>0.095</td>
<td>78.3</td>
</tr>
</tbody>
</table>

Substituting (Eq. 24) into (Eq. 23), we obtained the equation of the kinetic dependence of the change in the concentration of oil in the seed (rapeseed):

$$\bar{c}(\tau) = c_0 - \frac{1}{n} c_{p,0}(1 - e^{-k\tau}).$$

(25)

The graphical dependence constructed by the formula 25 is shown in Fig. 3; for microwave field generator power: 127, 225, 425 W.

To identify the diffusion equation with the internal source of the target component according to (18), it was necessary to have information about the initial (approximate) values of the coefficients included in Eqs. 16a and 20. The values of the coefficients $D_{ef}$ and $\beta$ were also determined from (Eq. 10, 11, 12).

To determine (in the first approximation) the quantitative value of the internal source of liquid phase removal under the action of the microwave field $q_m(N)$ according to the experiment, we used Eq. 20.

Restricting ourselves to the first term of the series (as the series converges rapidly (Eq. 19) and making certain transformations according to Lykov (1971), we obtained a simplified equation of the kinetics of the solid body extraction (by the analogy of the drying kinetics) with the source of the liquid phase extraction in the form:

$$-\frac{d\bar{c}(\tau)}{d\tau} = k_e \left(\bar{c}(\tau) - c_p(\tau)\right) - Q_m,$$

(26)

where:

- $k_e = \mu_1^2 \frac{R^2}{e_{ef}}$ extraction coefficient;
- $Q_m = q_m \mu_1^2 \left(1 + \frac{3}{W_{lm}}\right)$;
- $\mu_1^2 = 2.363 Blm - 1.398$;
- $c_p(\tau) = n(c_0 - c(\tau))$;
- $n = \frac{V_m}{V_p}$.

A solution of Eq. 26 under the initial condition:

$$\tau = 0; \bar{c} = c_0;$$

we got in the form:

$$\bar{c}(\tau) = B - (B - c_0) e^{-k_e(1+n)\tau},$$

(27)

where $B = \frac{1}{1+n} (nc_0) + \frac{q_m}{k_e}$.

From the analysis of dependence (Eq. 27) it followed that formally the value was equal to the value of the concentration of the target component of the solid phase in the set mode (finite, equilibrium concentration), the value of which was determined from equation (23):

$$\bar{c}_\infty = \bar{c}_0 - \frac{V_p}{V_m} c_{p,0} = B,$$

(28)

where $\bar{c}_{p,0}$ – final (established) value of the concentration of the extractant (determined from the graph (Fig. 3) of the kinetics of the extractant), kg m$^{-3}$.

Figure 3. Changing the concentration of rapeseed meal during extraction in alcohol at microwave generator power: 1 – 127 W; 2 – 225 W; 3 – 425 W.
The magnitude $k_e(1 + n)$ was the inverse of the transient constant time, which was determined by the exponential curve and could have been estimated from the properties of the exponential (20) (acceleration curves of the dynamic object).

To reduce the influence of random and systematic errors that occur during the experiment on the result of the determination of kinetic coefficients, it was advisable to use the data sets defined by empirical dependencies, in particular (Eq. 24, 25).

To use the Levenberg-McVard algorithm to find unknown mass-exchange coefficients (the value of the internal source of liquid phase removal under the action of the microwave field $q_m(N)$, and the mass-exchange coefficients $\beta, D_{ef}$) in Eq. 20 according to the data obtained from empirical dependence, we first approximated $\mu_1$–$\mu_5$ transcendental characteristic Eq. 21.

For $\mu_1$–$\mu_5$, after the numerical experiments with different approximating mathematical structures, the dependence with the highest level of multiple correlations of the form was determined:

$$
\mu = b_0 + \frac{B_i m}{b_1 + b_2 B_i m},
$$

(29)

The values of the coefficients of dependence (Eq. 29) were determined in Statistica 10 package and are shown in Table 2.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$b_0$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>R (multiple correlation coefficient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>0.21</td>
<td>0.8</td>
<td>0.74</td>
<td>0.9998</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>3.12</td>
<td>2.52</td>
<td>0.60</td>
<td>0.9999</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>6.26</td>
<td>4.79</td>
<td>0.57</td>
<td>0.9999</td>
</tr>
<tr>
<td>$\mu_4$</td>
<td>9.41</td>
<td>7.33</td>
<td>0.55</td>
<td>0.9999</td>
</tr>
<tr>
<td>$\mu_5$</td>
<td>12.55</td>
<td>10.09</td>
<td>0.52</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

Substitute dependence (Eq. 29) instead of $B_i m$ complex $\frac{\beta R}{D_{ef}}$ and thus determined the dependences $\mu_1$–$\mu_5$ substitute for dependence (Eq. 20). Using the built-in genfit function in the Mathcad mathematical package that implements the Levenberg-McWardt algorithm, we determined the values of $q_m, \beta, D_{ef}$ in which dependence (Eq. 20) would most accurately describe the set of defined data sets by dependency (Eq. 25). The on-screen form of the calculation example was presented in Fig. 4.

![Figure 4. Screen form for determining coefficients in Mathcad mathematical package](image-url)
In Fig. 5, the points presented were determined by empirical dependence (Eq. 25) and the curves obtained by analytical dependence (Eq. 20) \( (q_m, \beta, D_{ef}, \) determined by the genfit function in MathCAD). In Fig. 6, a comparison was presented.

Thus, the obtained analytical dependence of the change in the concentrations of the substance over time (Eq. 20) in the presence of the values determined as a result of the identification procedure \( (q_m = 0.108; D_{ef} = 3.685 \times 10^{-10}; \beta = 6.683 \times 10^{-7}) \) coincided quite accurately with the experimental dependences (Figs. 4 and 5). This allowed further research to use the known solutions of Eq. 16a with boundary conditions (Eq. 17) to analyze the course of extraction processes in new plants, as well as to determine the rational modes of operation of existing extractors and optimize processes by speed or minimization of energy consumption.

**Figure 5.** Changing the concentration of rapeseed meal during alcohol extraction at microwave generator power: 1 – 127 W; 2 – 225 W; 3 – 425 W (■, ●, ▲ – empirical points and analytical curves obtained by dependence (20) and identified coefficients)

**Figure 6.** Dependence of oil concentration in solution on time during the extraction of rapeseed meal in a microwave field with power: 1 – 425 W; 2 – 225 W; 3 – 127 W; (–) analytical curves obtained by dependence (20) and the identified coefficients; according to experimental data (–––)

**Conclusions**

1. The presented mathematical model of the process of extraction of vegetable raw materials under the action of a microwave field expands the physical understanding of the processes of heat and mass transfer in the system 'solid-liquid', allows quantifying the parameters of the process.

2. The proposed method of parametric identification of the mathematical description of the dynamics of extraction allows determining the kinetic coefficients of the process from one experimental experiment on the current installation.

**Conflict of interest**
The authors declare that they have no conflict of interest. No funds from the public or private sector were used for this research. The authors covered all expenses.

**Author contributions**
VB, RK – writing a manuscript;
SG, OL – conducting experimental research;
BK – editing and approval of the completed manuscript;
VG – design and manuscript design.
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