

Theory of oscillations performed by tools in spiral potato separator

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Abstract. The aim of this study is to provide for the intensification of the process of removing impurities from the potato heap by oscillating devices, which takes place, when potato tubers are cleaned in the spiral separator during their lifting. The authors have devised a new design of the spiral separator, which can be used in the process of lifting potatoes from the soil. It comprises cantilever cleaning spiral springs, which in their simultaneously rotary and oscillatory motion clean and transport the potato tubers with good quality. However, the removal of impurities from the cleaning area and the cleaning of the side surfaces of potato tuber bodies from the stuck soil can be significantly more effective, if the spiral springs perform their oscillatory motions in the longitudinal and vertical plane during their operation more intensively. In order to achieve that, the authors have worked out a new theory of the oscillatory motion of the cleaning spiral working under the action of the potato heap. That promotes the intensification of the process of cleaning potato bodies from the stuck soil. The authors have generated a new partial differential equation, which describes the oscillations of the cantilever cleaning spiral. The PC-assisted numerical solving of the said differential equation and further numerical modelling have made it possible to obtain the analytic expressions of the relations between the change in the helix pitch distance of the cleaning spiral and its deformation, in particular, the simultaneous longitudinal extension and transverse bending. That result, in its turn, provided for finding the maximum limit amount of the above-mentioned deformation under the condition that the potato tubers may not fall through the spaces between the spiral turns, taking into account the design and kinematic parameters of the cleaning spiral itself, the material it is made of and the process-dependent mode of operation of the separator. As a result of the PC-assisted numerical modelling, it has been found that the total deflection of the spiral on its length varies within the range of 0 to 0.05 m under the following parameters: angular velocity of rotation of the spiral $\omega = 30 \text{ rad s}^{-1}$, density of the material of the spiral $\rho = 7,700 \text{ kg m}^{-3}$, elastic modulus $E = 2 \cdot 10^{11} \text{ Pa}$, cleaning spiral bar stock radius $r = 8.5 \text{ mm}$ and a uniform load intensity of $1,000 \text{ N m}^{-1}$. Such variation ensures the good quality of cleaning and transportation of potato tubers. According to the results of the PC-assisted numerical calculations, the helix pitch distance of a cleaning spiral with the above parameters and the original helix pitch distance $S = 48 \text{ mm}$, due to the spiral's deformation under the proposed transverse oscillations, can change up to 54 mm, which makes impossible for potato tubers to fall

out of the cleaning unit. Experimental studies fully support the results from the theoretical calculations to determine the displacement of the ends of the cleaning unit's spiral springs.

Key words: cleaning unit, deformation, oscillatory motion, potato, soil.

INTRODUCTION

One of the principal requirements in the potato harvesting is to ensure the good quality cleaning of the potato heap from soil and plant impurities as well as the cleaning of individual potatoes from the stuck soil, reduction of their damage and loss rates. Meanwhile, the high quality cleaning of potato tubers from impurities is possible only in case a considerable amount of the soil and other components of the lifted potato heap is immediately separated from the tubers at the beginning of the heap transportation, or right after the potato bed is lifted from the soil by the lifting implement. In that case, a significant amount of the soil and plant debris will not enter the further parts of the potato harvester with its cleaning devices, which will provide for the adequate quality of harvesting.

However, the existing potato heap cleaning tools employed in the state-of-the-art potato harvesters insufficiently meet the above-listed requirements. Potato harvesting often takes place on rather damp and soft soils – as a result, it is a frequent occasion in the existing cleaning tools that the separating spaces get clogged with sticky wet soil and plant debris, which significantly reduces the capability of the potato heap cleaning units to reject the impurities and, consequently, the quality of the cleaning of potato tubers.

The machine that is most compliant with the above conditions of cleaning potato tubers from impurities during their lifting is the worked out new separator of the spiral type, the structural layout of which is presented in Fig. 1.

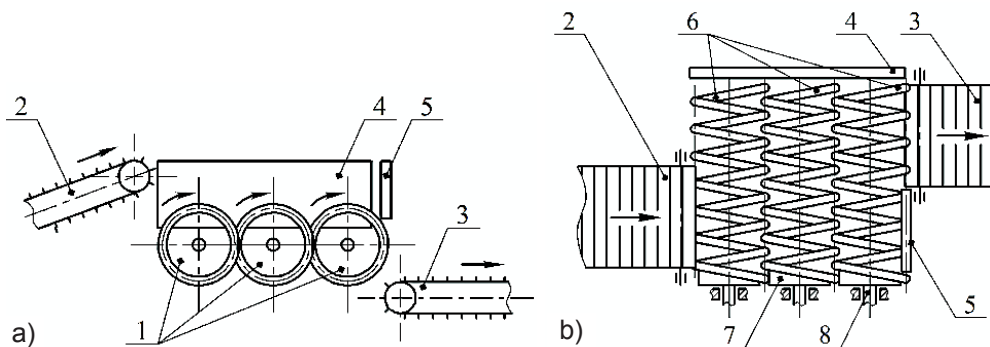


Figure 1. Spiral separator of potato heap lifted from soil: a – side view, b – top view; 1 – spirals; 2 – feeding conveyor; 3 – discharge conveyor; 4 – side protective apron; 5 – face protective apron; 7 – hub; 8 – drive shaft (Bulgakov et al., 2019).

As regards its structure, the cleaning unit is designed in such a way that the process of breaking the potato heap just dug out from the soil, dividing it into separate components and removing the soil impurities and plant debris out of the cleaning space takes place on the three cleaning spirals 1 installed in series. The spirals 1 are cantilevered with their ends on one side attached to the hubs 7 mounted on the drive

shafts 8, which generate their rotary motion in the same direction at a pre-set angular velocity of rotation. Their second (free) ends 6 can perform oscillatory motion in the longitudinal and vertical plane under the action of the variable load. The heap of the potato tubers dug out from the soil is fed with the use of the feeding conveyor 2, while the evacuation of cleaned potato tubers from the spirals 1 is performed by the discharge conveyor 3. In order to prevent the loss of potato tubers during the work process of their cleaning from soil impurities and plant debris, the rectangular end face 4 and side 5 protection shields are installed around the spirals 1. The cantilever cleaning spirals 1 are installed with mutual overlapping, which facilitates their self-cleaning from the stuck wet soil.

The efficiency of the self-cleaning of the cleaning spirals 1 is provided by their oscillatory motion, which produces the variable deformation of the spirals 1 themselves, that is, their longitudinal extension and transverse bending. Such conditions result in the spacing between any adjacent turns of the spirals 1, i.e. the helix pitch distance, constantly changing, which promotes detachment (and compaction) of the stuck soil in the turn-to-turn space followed by its sieving down and out of the cleaning unit. Thus, in the proposed potato heap cleaning unit not only the self-cleaning of the cleaning spirals 1 takes place, but also the active separation of soil impurities and plant debris through the spaces between the turns of the spirals 1 as well as between the spirals 1 themselves.

Since the spirals 1 are attached, via the hubs 7, to the drive shafts 8, they perform rotational motions and, therefore, the spirals 1 entrain with their turns the bodies of potato tubers and soil clots and start transporting them in both the longitudinal and radial directions. However, in view of the present large gaps (between the turns of each spiral 1 and also the space between the spirals 1 themselves), the soil particles in great quantities are immediately sieved down and out of the separator.

Hence, the surface formed by the three cleaning spirals in the discussed unit for cleaning potatoes from impurities features significant sieving openings, also the said surface is active in view of the rotary motion of the spirals and the spirals generate the motion of the components of the heap fed for cleaning simultaneously in two directions, that is, the radial and axial ones. Moreover, the cantilever mounting of the cleaning spirals implies that their free ends perform oscillatory motion in the longitudinal and vertical plane, which not only promotes the development of favourable conditions for the efficient agitation of the potato heap, but also generates the periodic changes in the turn-to-turn spacing and that in effect provides the conditions for the positive entrainment of strong soil bodies and plant debris and their removal out of the cleaning area. At the same time, the potato tubers may not become entrapped by the spiral turns or sustain damage. Finally, the mutual overlapping of the spirals and their oscillatory motion under the action of the variable load can result in the eccentricity of the ends of the spirals, which promotes the generation of repetitive potato tuber shock motions and that, in its turn, facilitates their effective cleaning from the stuck soil.

The problem of the development and research of potato heap separators is discussed in the papers (Petrov, G., 1984; Feller et al., 1987; Holland-Batt, 1989; Misener & McLeod, 1989; Peters, 1997; Krause & Minkin, 2005; Bishop et al., 2012; Ichiki et al., 2013; Guo & Campanella, 2017; Wang et al., 2017; Ye et al., 2017; Wei et al., 2017). However, as stated above, the cleaning tools mentioned in those papers have one essential deficiency – their cleaning gaps are clogged by soil and plant debris, when operating on soils with an increased water content, which significantly reduces the separating capacity of such tools. The authors have devised the spiral separator

(Bulgakov et al., 2018a, 2018b, 2018c; 2019) that is capable of not only efficiently cleaning potato tubers from impurities, but self-cleaning from the stuck soil as well. The authors have also carried out its testing and initial research. In order to optimise the design and kinematic parameters of the new cleaning unit, it is necessary to carry out the theoretical investigation of the operation of its tools.

The investigation of the beam deflection curve can be done with the use of the differential equation for the case of loading by a free system of forces and a moment at the end (Vasilenko, 1996) or on the basis of strain compatibility equations (Chelomey, 1981). In a simplified form, such calculations of deflection can also be performed with the use of the equations obtained in (Tymoshenko, 1959). But, in the above cases, beams with constant cross-sections and constant properties are assumed. For the spiral spring, the moment of inertia of which is a function of the axial coordinate and time, these methods do not provide any new information. Therefore, for the purposes of this study, it would be most reasonable to apply the Bernoulli-Euler differential equation of the transverse flexural oscillations of a deformed beam, which comprises the relation between the curvature and the deflection and the differential equation of static beam deflection by J. Bernoulli together with the dynamic member added to the equation by L. Euler to take account of the transverse inertia forces. In case of free oscillations, the right-hand member of the equation is equal to zero. In case of an external perturbing force acting on the beam, the right-hand member of the equation assumes the form of the said perturbing force function. In case of a load uniformly distributed in time and along the axial coordinate, the right-hand member of the equation assumes the form of the uniformly distributed load.

The aim of the study is to provide for the intensification of the process of removing impurities from the heap by vibrating tools in the process of cleaning potato tubers by the spiral separator at the stage of their lifting.

MATERIALS AND METHODS

In the development of the theory of oscillations of the cleaning tools in the spiral potato separator, the methods of the modelling and the strength of materials have been used, in particular, the theory of the oscillations of elastic and deformed beams on the basis of the Bernoulli-Euler theory, the application of the Krylov special functions as well as the methods of programming and numerical modelling.

In view of the fact that the oscillatory motion of cantilever cleaning spirals plays an important part in the work process of cleaning potato tubers from soil and plant debris, the need arises first of all to investigate the said oscillatory process analytically.

For that purpose, it is necessary first of all to generate a mathematical model of the oscillatory process performed by the spirals under the action of a variable load.

The first step is to work out the equivalent schematic model of the oscillations performed by the spiral attached to the drive shaft in cantilever mode, which rotates and is under the action of the external load produced by the potato heap situated on it (Fig. 2).

The equivalent schematic model (Fig. 2) features the cleaning spiral in two positions: unstrained condition, where its longitudinal axis coincides with the OZ axis, and the deformed condition, where its longitudinal axis undergoes a deflection under the action of the variable distributed load designated $\tilde{q}(z, t)$. The said load is produced by

the potato heap continuously fed by the feeding conveyor onto the surface of the cleaning spirals. In the schematic model under consideration, the parameter S is shown, which is the helix pitch distance of the unstrained spiral and it has the same value all along the whole length of the spiral. The schematic model features also $S(z, t)$ – the helix pitch distance of the spiral that undergoes deformation under the action of the of the external load. This deformed spiral pitch distance varies both along the length of the spiral itself and with time due to the spiral’s oscillatory motion. The deflection of the spiral’s longitudinal axis designated $W(z, t)$ is also variable along the length of the spiral’s axis (z coordinate) and depends on time (parameter t) as a result of the spiral’s oscillatory motion. R – the spiral’s radius. ω – angular velocity of the spiral’s rotary motion about its longitudinal axis (sense of rotation is shown by arrow).

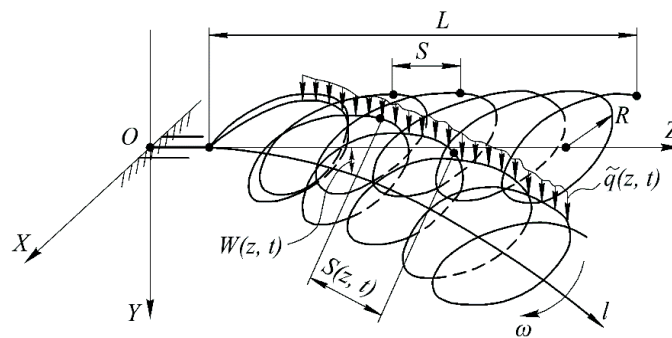


Figure 2. Equivalent schematic model of oscillations of spiral spring in potato heap separator (Nowak et al., 2019).

This investigation of the transverse oscillations of the cantilever cleaning spiral is carried out in the absolute coordinate system $OXYZ$, in which the axis OZ is directed along the longitudinal axis of the unstrained spiral, the axis OY is directed downwards and the axis OX – perpendicular to the plane OYZ . The longitudinal axis of the deformed spiral is designated Ol .

Thus, there are good reasons to investigate theoretically the process of oscillation of the potato heap cleaning unit spiral considering it as an elastic beam designed in the form of a cylindrical spring with a radius of R , a winding pitch of S and a helix angle of γ .

It is to be noted already now that under the operating conditions with the load varying along the length of the spiral and in view of the variable properties of the spiral with regard to its compliance to the flexural and torsional deformations, the helix pitch distance parameter S becomes variable along the length and this variation is not in accordance with the plane-sections hypothesis.

RESULTS AND DISCUSSION

In order to simplify the investigation at this point and taking into account the rigidity of the spiral, the curved beam is replaced by a cantilever beam with the presented parameters.

Hence, as a first approximation, there is a possibility to describe the flexural oscillations of the spiral (considered, by convention, as a cantilever beam) by the differential equation of the following form (Chelomey, 1981):

$$\frac{\partial^2}{\partial z^2} \left(E \cdot I_R \frac{\partial^2 W}{\partial z^2} \right) + \rho \cdot F \frac{\partial^2 W}{\partial t^2} = \tilde{q}(z, t) \quad (1)$$

where W – deflection of the spiral's longitudinal axis; $\tilde{q}(z, t)$ – distributed and time-variant load of the potato heap on the cleaning spiral; ρ – specific gravity of the spiral's material; F – area of cross section of the cleaning spiral bar stock; E – Young modulus; I_R – reduced moment of inertia of the spiral.

In the classical bending theory, a hypothesis is admitted that plane sections retain their planarity also after the deflection, while the flexural deformation is directed normally to the longitudinal fibres of the beam. That is, for the purpose of simplifying the theoretical investigation in the case under consideration, an assumption is made that pure bending takes place.

The result of solving the differential Eq. (1) is the relations between the deflection of the cantilever beam and its length for the cases of free and forced oscillations (Dolgov, 1996; Grote & Antonsson, 2008).

The next step is to perform the necessary mathematical transformations. For example, the general solution of the differential Eq. (1) can be represented by the following sum of functions:

$$W(z, t) = W^*(z, t) + \tilde{W}(z, t), \quad (2)$$

where $W^*(z, t)$ – general solution of the homogeneous differential equation that has the following form:

$$\frac{\partial^2}{\partial z^2} \left(E \cdot I_R \frac{\partial^2 W}{\partial z^2} \right) + \rho \cdot F \frac{\partial^2 W}{\partial t^2} = 0 \quad (3)$$

$\tilde{W}(z, t)$ – partial solution of the original differential equation with the right-hand member.

In this case, the starting and limit conditions are as follows:

at $z = 0, t = 0,$

$$W(z, t) = 0, \quad \frac{\partial^2 W}{\partial z^2} = 0 \quad (4)$$

at $z = L,$

$$E \cdot I_R \frac{\partial^2 W}{\partial z^2} = 0, \quad \frac{\partial}{\partial z} \left(E \cdot I_R \frac{\partial^2 W}{\partial z^2} \right) = 0. \quad (5)$$

The partial solution $\tilde{W}(z, t)$ can be written in the form of a relation between the deflection and the length of the cleaning spiral (Dolgov, 1996; Grote & Antonsson, 2008). Thus, the following is obtained:

$$\tilde{W}(z, t) = \left[q \cdot L^2 \left(\frac{z^4}{L^2} - \frac{2z^3}{L} + 6z^2 \right) \right] \cdot (24E \cdot I_R)^{-1} \quad (6)$$

where L – length of the spiral.

The general solution of the homogeneous differential Eq. (3), following the Fourier method, can be written in the form of the product of the longitudinal coordinate function $Z(z)$ and the time function $T(t)$. That is:

$$W^*(z, t) = Z(z) \cdot T(t). \quad (7)$$

It is assumed that the frequency of the spiral's natural oscillations is k times greater than its rotation frequency. In this case, k – multiplicity of oscillations. Then, it can be assumed that:

$$T(t) = \cos(\omega k t). \quad (8)$$

The first factor in the expression (7) can be written in the form of the Krylov functions (Ananyev, 1965; Dolgov, 1996):

$$Z(z) = AS(\lambda z) + BT(\lambda z) + CU(\lambda z) + DV(\lambda z), \quad (9)$$

where $S(\lambda z)$, $T(\lambda z)$, $U(\lambda z)$ and $V(\lambda z)$ – special functions introduced into the mechanics by A.N. Krylov; A , B , C and D – constants determined by substituting the general solution into the limit conditions; λ – spectrum of oscillation frequency eigenvalues.

Hence, taking into account the expressions (2), (7), (8) and (9), the general solution of the differential Eq. (1) assumes the following form:

$$W = \left[AS(\lambda z) + BT(\lambda z) + CU(\lambda z) + DV(\lambda z) \right] \cdot \cos(\omega \cdot k \cdot t) + \left[qL^2 \left(\frac{z^4}{L^2} - \frac{2z^3}{L} + 6z^2 \right) \right] \cdot (24E \cdot I_R)^{-1}. \quad (10)$$

Further, it is necessary to determine the reduced second moment of area I_R of the spiral, which is part of the expression (10) and is a value that varies along the length of the spiral, because depends on the angular parameter ψ , which defines the current length of the spiral along its longitudinal axis z . In order to find it, the following expression can be used (Svetlinsky, 1967):

$$I_R = \frac{\pi \cdot r^4}{4} \left\{ \sin \gamma \left[1 + (1 + 2\nu) \sin^2 \psi \cdot \cos^2 \gamma \right]^{-1} \right\}, \quad (11)$$

where r – radius of the bar stock, from which the spiral is wound; ν – Poisson's constant of the material, from which the spiral is made.

In case the angular parameter of the spiral is represented by the following expression:

$$\psi = \psi_o + \omega t = \frac{2\pi \cdot z}{S} + \omega t \quad (12)$$

where ω – angular velocity of the spiral's rotary motion, the relation between the reduced second moment of area I_R of the spiral cantilever beam, on the one hand, and the length (z coordinate) and the time t , on the other hand, is obtained at the constant values of the helix pitch distance $S = \text{const}$ and the angular velocity $\omega = \text{const}$, that is:

$$I_R = \frac{\pi \cdot r^4}{4} \left\{ \sin \gamma \left[1 + (1 + 2\nu) \sin^2 \left(\frac{2\pi \cdot z}{S} + \omega t \right) \cdot \cos^2 \gamma \right]^{-1} \right\}. \quad (13)$$

Upon carrying out PC-assisted numerical calculations (13), the graphical representation is obtained for the relation between the reduced second moment of area I_R of the spiral and its longitudinal coordinate z (Fig. 3).

With the use of the data presented in Fig. 3, the differential Eq. (10) that represents the relation between the process of oscillations of the cantilever spiral and the spiral's design parameters and material properties can be numerically solved.

In view of the fact that the coefficients at the Krylov functions have certain numerical values for each position of the spiral, its PC-assisted solving can be done in the form of diagrams, taking into account the following design, kinematic and force parameters: angular velocity of rotation of the spiral $\omega = 30 \text{ rad s}^{-1}$; density of the material (spring steel), from which the spiral is made, $\rho = 7,700 \text{ kg m}^{-3}$; elastic modulus $E = 2 \cdot 10^{11} \text{ Pa}$; spiral bar stock radius $r = 8.5 \text{ mm}$; under the action of a uniformly distributed load with an intensity of $1,000 \text{ N m}^{-1}$ at the points of time: 1) 0 s; 2) 0.05 s; 3) 0.25 s (Fig. 4, 5).

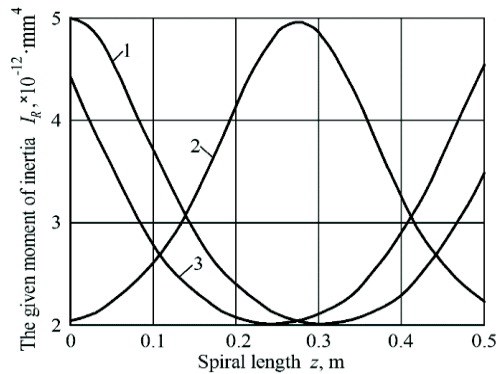


Figure 3. Variation of reduced second moment of area I_R of cantilever beam along length of spiral with spring diameter $d = 17 \text{ mm}$, helix pitch distance $S = 48 \text{ mm}$, helix angle $\gamma = 7^\circ$ and angular velocity of rotation $\omega = 30 \text{ rad s}^{-1}$ at points of time: 1) $t = 0 \text{ s}$; 2) $t = 1 \text{ s}$; 3) $t = 2 \text{ s}$.

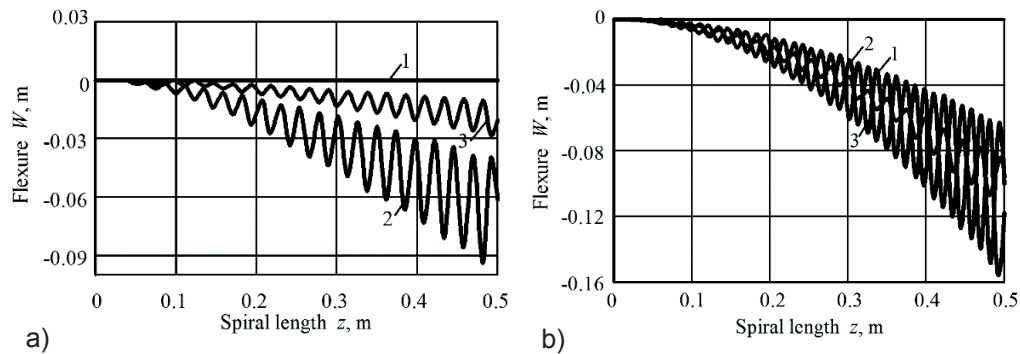


Figure 4. Relation between deflection W of axis at free end of separator spiral and its length z at points of time: 1) $t = 0 \text{ s}$; 2) $t = 0.05 \text{ s}$; 3) $t = 0.25 \text{ s}$: a – in case of free flexural oscillations; b – under action of uniformly distributed load.

The diagram in Fig. 4 a represents the transverse oscillations of the unloaded spiral, which take place under the action of solely the weight of the spiral itself distributed along the longitudinal axis. In this case, the deflection varies from 0 to 0.1 m. Under the action of the distributed load produced by the potato heap (Fig. 4 b), the oscillations take place about the bent axis of the spiral. In this case, the deflection of the spiral axis changes within the range of 0 to 0.16 m.

When both the oscillations of the unloaded spiral and the oscillations under the action of the uniformly distributed load are taken into account, the spiral axis deflection varies from 0 to 0.25 m.

In order to determine the variation of the cleaning spiral pitch distance S in the process of its deformation during oscillations, the spiral's flexure geometry is to be analysed in detail. For that purpose, the equivalent schematic model of the spiral axis bending is to be generated (Fig. 6). It is assumed that, conventionally, the spiral flexure can be analysed considering it as an arc with a respective curvature.

For the purpose of describing the longitudinal and transverse deformation of the spiral, the plane Cartesian coordinate system OZY is introduced with its origin (point O) situated at cantilever spiral attachment point, its OZ axis directed horizontally to the right and the OY axis directed downwards.

At the same time, it is assumed that the longitudinal axis of the unstrained (straight) spiral is aligned with the axis OZ , the longitudinal axis of the bent spiral is aligned with the curve OL . In order to investigate the longitudinal (extension) and transverse (bending) deformation of the spiral axis, two arbitrary adjacent turns of the straight spiral can be taken. The turn on the left is assigned the number i , the turn on the right – the number $i + 1$, while the centre of the i -th turn is the point A_0 , the centre of the $i + 1$ -th turn – the point B_0 , both situated on the OZ axis with the coordinates $A_0(z_i; 0)$ and $B_0(z_{i+1}; 0)$, respectively. It is obvious that the distance between the points A_0 and B_0 is equal to $A_0B_0 = z_{i+1} - z_i = S$, where S – helix pitch distance of the straight spiral, $S = \text{const}$.

Further, the longitudinal deformation and then the transverse deformation of the spiral have to be analysed. Following the longitudinal deformation of the spiral, the point A_0 will move along the OZ axis to some point A_1 , the point B_0 – to some point B_1 with the coordinates $A_1(z_i + \Delta_i); 0$ and $A_1(z_{i+1} + \Delta_{i+1}); 0$, respectively, where Δ_i and Δ_{i+1} – displacements of the points A_0 and B_0 , respectively, caused by the longitudinal deformation of the spiral. Following the transverse deformation of the spiral, the point A_1 will move, due to the deflection, to the point A , the point B_1 – to the point B . These

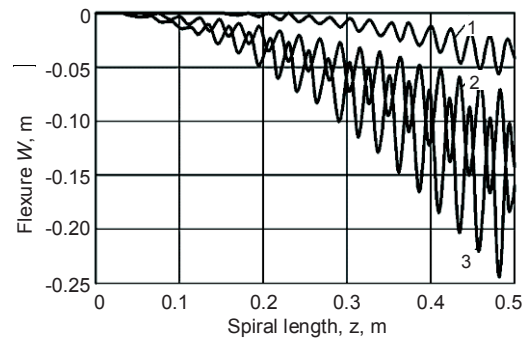


Figure 5. Relation between total deflection W of axis of spiral and its length z in case of natural flexural oscillations and under action of uniformly distributed load: at points of time: 1) $t = 0$ s; 2) $t = 0.05$ s; 3) $t = 0.25$ s.

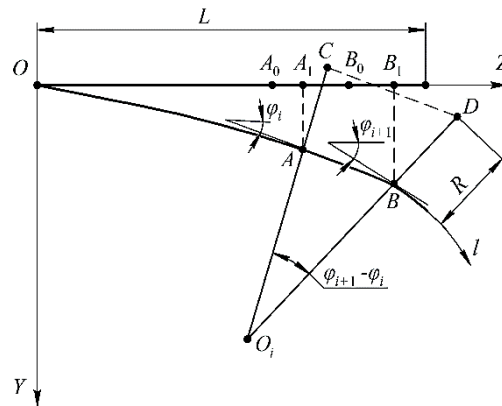


Figure 6. Equivalent schematic model of cantilever elastic spiral bending.

points are situated on the curved axis Ol , and also, the point A_1 is displaced along the axis OY by the amount of deflection W_A , the point B_1 – by the amount of deflection W_B .

Thus, the points A and B will become the centres of the turns with the numbers i and $i + 1$, respectively, after the simultaneous longitudinal and transverse deformations, and their coordinates will be $A(z_i + \Delta_i; W_A)$ and $B(z_{i+1} + \Delta_{i+1}; W_B)$, respectively.

The distance between the points A and B is determined as follows:

$$AB = \sqrt{(Y_B - Y_A)^2 + (Z_B - Z_A)^2} = \sqrt{(W_B - W_A)^2 + (S + \Delta_{i+1} - \Delta_i)^2}. \quad (14)$$

The curve that passes through the points A and B , represents the flexure of the spiral's longitudinal axis. Tangents to the curve have to be drawn at these points. The angles between the tangents and the horizontal are φ_i and φ_{i+1} , respectively, and they represent the angular displacement of the cross-sections with respect to their initial positions. The angular displacements of the cross-sections are found with the use of the following expressions (Ananyev, 1965):

$$\varphi_i = \arctan \frac{\partial W}{\partial z} \text{ at } z = Z_A; \quad \varphi_{i+1} = \arctan \frac{\partial W}{\partial z} \text{ at } z = Z_B. \quad (15)$$

Assuming that the normal lines drawn through the points A and B meet at the point O_i , the distances from that point to the points A and B are the curvature radii of the curve AB at the points A and B , respectively.

Further, the following designations are introduced: $O_iA = \rho_i$, $O_iB = \rho_{i+1}$. Then, according to (Ananyev, 1965), the following is arrived at:

$$\rho_i = \frac{1}{\frac{\partial^2 W}{\partial z^2}} \text{ at } z = Z_A, \quad \rho_{i+1} = \frac{1}{\frac{\partial^2 W}{\partial z^2}} \text{ at } z = Z_B \quad (16)$$

As is seen in Fig. 6, the angle between the above-mentioned curvature radii is equal to $\varphi_{i+1} - \varphi_i$.

If the normal lines O_iA and O_iB are extended by the length of the spiral's radius $R = AC = BD$, that will result in obtaining the final (upper) points C and D of the deformed i -th and $i + 1$ -th turns, respectively, of the operating (separating) surface, on which the process material is situated. As is seen in Fig. 6, the coordinates of the points C and D in the assumed frame of reference are equal to:

$$\begin{aligned} Y_D &= W_B - R \cos \varphi_{i+1}, & Y_C &= W_A - R \cos \varphi_i, \\ Z_D &= Z_B + R \sin \varphi_{i+1}, & Z_C &= Z_A + R \sin \varphi_i. \end{aligned} \quad (17)$$

The distance between the points C and D represents the distance between the adjacent turns, i.e. the pitch distance between the turns in the upper part of the spiral after the deformation of the spring.

Then, the said pitch distance CD is equal to:

$$CD = \sqrt{(Y_D - Y_C)^2 + (Z_D - Z_C)^2}, \quad (18)$$

or, taking into account the expression (17), the following is obtained:

$$CD = \sqrt{[W_B - W_A - R(\cos \varphi_{i+1} - \cos \varphi_i)]^2 + [S + \Delta_{i+1} - \Delta_i + R(\sin \varphi_{i+1} - \sin \varphi_i)]^2}. \quad (19)$$

The problem under investigation presumes the following requirement to the acceptable values of the winding pitch: the potato tubers may not fall through the turn-to-turn spaces. Therefore, it is necessary to ensure that $CD \leq [S_{max}]$, where $[S_{max}]$, –

maximum acceptable value of the winding pitch after the deformation of the spiral, which is defined by the geometric parameters of the potato tubers.

Taking into account the above-mentioned in equation and the expression (19), the criterion is obtained for the potato tubers not falling through the turn-to-turn space with due account for the design and kinematic parameters of the cleaning spiral, the material, from which it is made, and the process modes of operation of the potato heap cleaning unit.

By employing the cosine theorem and analysing the triangle O_iCD (Fig. 6), the equivalent expression can be formulated for determining the winding pitch of the spiral during its deformation of flexure with simultaneous extension. Then, taking into account the above-mentioned in equation, the equivalent criterion is obtained for the potato tubers not falling into the turn-to-turn space:

$$CD = \sqrt{(\rho_i + R)^2 + (\rho_{i+1} + R)^2 - 2(\rho_i + R)(\rho_{i+1} + R)\cos(\varphi_{i+1} - \varphi_i)} \leq [S_{\max}]. \quad (20)$$

Thus, the obtained analytic expression (20) makes it possible to model the dynamics of the cantilever tool (cleaning spiral) during its performance of the work process with the chosen design parameters and modes of operation, subject to the requirement of the potato tubers not falling from the separating surface, under the action of a variable load and taking into account the variation of the second moment of area of the elastic cantilever beam with time and along its length.

Taking into account the obtained expressions (4), (5), (10), (15) and (16), it is possible to generate the formulae for determining the angular displacement φ_i of the spiral's cross-section at an arbitrary point z_i on the length of the spiral and the respective curvature radius ρ_i of the spiral's axis at the said point. Thereafter, on the basis of these formulae, it is possible to obtain the specific values of the angular displacement φ_i of the spiral's cross-section and the curvature radius ρ_i at a specific point along the length of the spiral.

Then, at the pre-set radius R of the spiral, it is possible to determine with the use of the formula (20) the pitch distance of the spiral during its flexural deformation simultaneously with the extension along its longitudinal axis. However, it is to be noted that the expressions for finding the first-order and especially the second-order derivatives of the expression (10) that determines the deflection at an arbitrary point along the length of the spiral are rather bulky, therefore, they are better calculated with the use of the PC.

Still, as a result of the completed analysis, it is possible to use the formula (20) for obtaining the maximum permissible helix pitch distance S between the adjacent turns of the spiral subject to the requirement of the potato tubers not falling through the spaces between those turns.

But, in case only a modest load is fed onto the cleaning spiral or its stiffness is sufficiently high, the variation of its pitch distance will be insignificant (due to the small deflection of the spiral's longitudinal axis). In such cases, the oscillations of the spiral will take place without any entrapping of the potato tubers or their falling inside the spiral spring.

Using the potato cleaner that was designed and developed by us, we have conducted experimental field studies involving the new spiral separator for potato tubers on a specially-developed and manufactured potato separator. Fig. 7 shows the overall view of the experimental potato separator.

The experimental studies were carried out in the course of harvesting potatoes from one row of the Ukrainian potato, *Lugovskaya*. The potato field's physical and technological characteristics were as follows: the yield for the potato crop was 40.35 t ha^{-1} . The potatoes were planted in rows with a row interval of 0.7 m. The soil in which the potatoes were planted was black earth with a medium humus content and a medium loam content; the soil's humidity level was between 6–8% in the upper layer, and 12–16% at a depth of 10 cm and deeper; soil hardness at the tuber-carrying level was 0.3–0.5 MPa.

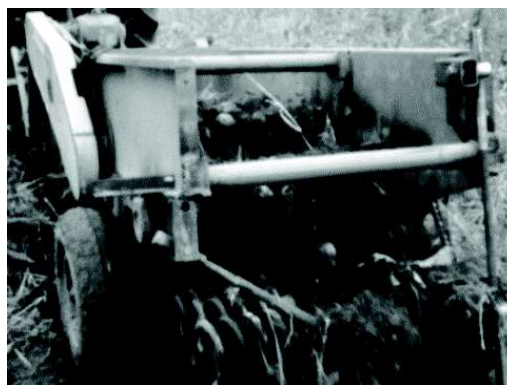


Figure 7. The potato separator during the field experiments.

The structural parameters of the spiral separator when the experimental studies were being conducted were as follows: the separator spiral's outer diameter was 133 mm; the pitch of the spiral's helix line was 25° ; the diameter of the spiral wire was 17 mm; the step of the spiral's helix was 48 mm; the overlap of the adjacent spirals was 6–8 mm; the minimum number of spirals was three; and the calculated length of the spiral for a row of those tubers that were being harvested was 500 mm.

The movement mode for the experimental potato separator when digging out potato tubers was chosen in the form of several variants: 0.53; 0.67; 0.83; and 1.11 m s^{-1} (or 1.91; 2.41; 2.99; and 4.0 km h^{-1} , respectively). At each speed of movement, experimental observations were carried out in five repetitions.

In the field experiments, second feeds of potato tubers were determined for the cleaning spirals, taking the depth from which to lift the tubers to be 27 cm; the width of the processed field strip was to be 55 cm, and the total mass of the potato heap was set as $1,300 \text{ kg m}^{-3}$.

During the course of the experimental potato separator's movement along the planting row of potatoes, the tuber-carrying layer of the soil was fed into the cleaner's spiral surfaces. Under the influence of the lifted tubers being fed into the spirals, the console ends of the cleaning spirals angled downwards, i.e. the longitudinal axes of each cleaning spiral were displaced downwards from their horizontal position by the value of W . These displacements, W , were measured during the experimental studies and the limits of the movements of the spiral ends were documented for each test.

To conduct the experimental field study in question, a multifactor experiment of type 2^{4-1} was implemented, with a repetition of three for each test.

The experimental tests provided an opportunity to study the effect of several factors on the outcomes of the spiral separator's operation. These involved the angular speed of its cleaning spirals, the eccentricity of its spirals on their drive shafts, the number of lifted tubers that were fed into the cleaner, and also the effect of the downward displacement, W , on the ends of the cleaning spirals. The output parameter was the amount of soil that was separated through the cleaning spiral, evaluated as a percentage. For each test in these experimental studies, the experimental potato separator and the tractor pulling it were reset to the initial parameters that corresponded to those factors

that were being monitored, and also to the movement speed that was being used. In view of this, samples were taken each time of the planted soil and other additives in the soil mixture that were passing through the cleaning spirals into prepared containers. The samples that were taken were then weighed on electronic scales with a precision of up to 1.0 g. The weighing results were entered into a table.

On the basis of the multifactor experiment that has been conducted, utilising the new spiral potato tuber separator that has been developed by us, statistical calculations were carried out on a PC using the data from the potato separator's performance results, and employing the main positions for correlation and regression analysis, and graphs were drawn up showing the dependencies between the relevant quality indicators.

It was in this way that dependencies were determined for the percentage of soil separation through the separator's cleaning spirals on the value of the downwards displacement W of the cleaning spirals (Fig. 8).

As can be seen from the graphs of dependencies that are presented here, an increase in the displacement W leads to a significant increase of the percentage of soil that is separated through the spiral. This is especially observable when increasing the moisture content in the soil layer that is having its potato tubers fed into the cleaner. In this way, if the soil layer's moisture content is 12.5% and the displacement W is at its maximum of 220 mm, the percentage of soil that can be separated through the cleaning spiral increases by 11%. This is explained by the fact that an increase of the displacement W in the cleaning spiral ends - and therefore also in the step S of those spirals - leads to the effective forced grabbing of soil additives and other soil clumps and their transfer outside the separator's borders. And yet such changes in the step S of the cleaning spirals do not lead to any crushing or breaking of the tubers, so there is practically no loss of or damage to potatoes. In the case of the tuber-carrying soil layer being drier, a significant spreading take place of the soil components through the cleaning spirals, along their longitudinal axes if these are horizontal. But if the aforementioned axes are inclined to the maximum displacement W , then the percentage of soil that can be separated through the cleaning spirals increases on average by 4–5%.

Therefore the values of the displacements of the cleaning spiral ends as determined by the experimental studies practically coincide with data for the theoretical calculations that was displayed in Fig 5. This means that the theoretically-calculated maximum value for the displacement of a cleaning spiral's end is 0.23m, which corresponds to the experimental data. Consequently, the theory of oscillations that is carried out by the cleaning spirals on a new spiral potato separator corresponds to the actual oscillation process.

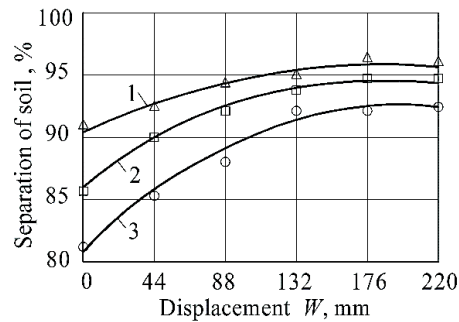


Figure 8. The dependence of soil separation on the displacement W of the cleaning spiral ends upon the following moisture contents of the mass that was fed into the separator: 1) 7.0%; 2) 9.5%; and 3) 12.5%.

CONCLUSIONS

1. The analytical mathematical model of the oscillations performed by the tools of the spiral potato heap separator has been developed, which has allowed to generate the differential equation of the transverse flexural oscillations of its cantilever cleaning spiral.

2. On the basis of solving the differential equation of the transverse flexural oscillations of the cleaning spiral, the analytic expressions that describe the law of the oscillatory process and the flexure of the spiral at an arbitrary point of time for any point on its longitudinal axis have been obtained.

3. Also, analytical dependences have been obtained for determining the variable pitch distance of the bent spiral at an arbitrary point of time and for any turn-to-turn space during the above-mentioned oscillatory process.

4. When the angular velocity of rotation of the spiral is equal to $\omega = 30 \text{ rad s}^{-1}$, the density of the material, from which the spiral is made, is equal to $\rho = 7,700 \text{ kg m}^{-3}$, the elastic modulus is $E = 2 \cdot 10^{11} \text{ Pa}$, the radius of bar stock is $r = 8.5 \text{ mm}$ and the uniformly distributed load of the potato heap on the spiral has an intensity of $1,000 \text{ N m}^{-1}$, the total deflection of the spiral along its length varies within the range of 0 to 0.25 m.

5. The analytic expressions have been obtained for the limitation of the maximum variation of the helix pitch distance of the cleaning spiral during its oscillations subject to the condition that potato tubers may not fall through the turn-to-turn spaces of the spiral, taking into account the design and kinematic parameters of the cleaning spiral, the material it is made of, the process modes of operation and the tuber sizes.

6. The PC-assisted numerical calculations have established that the helix pitch distance of the cleaning spiral with the above-listed parameters and the original helix pitch distance $S = 48 \text{ mm}$ can vary within the range of up to 54 mm due to the deformation during the considered transverse oscillations, which ensures the potato tubers not falling out of the cleaning unit.

7. The results of the experimental studies that have been conducted in regards to the oscillation process of the cleaning spirals on a new spiral potato separator all serve to confirm the calculated theoretical data.

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