E. Machinery and Fine Mechanics

TALLINNA TEHNIKAÜLIKOOLI
VÄITEKIRJAD

THESES
OF TALLINN TECHNICAL UNIVERSITY

ДИССЕРТАЦИИ ТАЛЛИННСКОГО
ТЕХНИЧЕСКОГО УНИВЕРСИТЕТА

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Determination of Residual Stresses in Coatings and Coated Parts

TALLINN 1994

DECLARATION

I declare that this thesis is my original, unaided work. It is being submitted for the degree of Doctor of Engineering of Tallinn Technical University, Estonia. It has not been submitted before for any degree or examination in any other university.

Signature of candidate: J Kõo

Date: 05.11.93

KOKKUVÕTE

Jääkpingete määramine pinnetes ja pindega detailides

Väitekirjas käsitletakse eksperimentaalsete meetodite väljatöötamist ja täiustamist jääkpingete ja -deformatsioonide määramiseks pinnetes ja pindega detailides.

Esitatakse üldine algoritm, mis võimaldab määrata jääkpinged kas pinde kasvamisel (pindamisel) või kahanemisel (eemaldamisel) aluse vabal pinnal või pinde liikuval pinnal mõõdetud deformatsiooniparameetrite järgi. On koostatud algoritmid jääkpingete määramiseks paksu mitmekihilise pindega mitmekihilistes ristkülikvarrastes, plaatides, silindrites ja kerades.

Täpsustatakse algpingete arvutust sirge ribaaluse, ümartraataluse, õhukeseseinalise sfäärilise aluse, õhukeseseinalise rõngasaluse ja kruvijoonelise ribaaluse deformatsiooniparameetrite järgi. Esitatakse õhukeseseinalise toru-aluse ja lahtise rõnga kujulise aluse deformatsiooniparameetrite mõõtmise meetodid algpingete määramiseks vastavalt galvaanilistes ja tampoongalvaanilistes pinnetes.

Fotoelastsusmeetodiga on kontrollitud jääkpingete jaotust sirge varrasaluse ühepoolses pindes. Termobimetallanaloogiat kasutades on näidatud, et ühepoolse pinde kasvamisel või kahanemisel paindub riba- või plaatalus sfääriliselt. Eksperimentaalselt on kontrollitud ühepoolse pindega kruvijoonelise ribaaluse puhta painde teooriat. On uuritud jääkpingeid paksudes galvaanilistes teraspinnetes.

ACKNOWLEDGEMENTS

I would like to thank all who have, in one way or another, contributed to this thesis and in particular, the following persons:

– Prof. I. A. Birger (Moscow), Prof. L. I. Dekhtyar (Kishinev), Prof. S.I. Ivanov (Samara), Dr. M. L. Kozlov (Lvov), Dr. A. A. Martynenko (Rovno), Prof. A. Peiter (Saarbrücken) and Prof. M. Perakh (Woodland Hills) for their interest in my work.

– Dr. S. Idnurm (Tallinn), Dr. H. Lille (Tartu), Dipl.-Eng. M. A. Meshcheryakov (Moscow), Prof. V. A. Smirnov (Moscow) and Dipl.-Eng. J. Valgur (Tartu) for their helpful collaboration.

CONTENTS

INTRODUCTION

Motivation and presentation of the thesis

In recent decades, strengthening and protecting new machine parts and rebuilding of worn-out parts by application of various coatings has been the accepted procedure. Almost all the coating technologies cause residual stresses and deformations in the coated parts.

It is well-known that residual stresses may have both detrimental and favourable consequences for the service characteristics of coated parts. The experiences range from the cracking of the coating up to the considerable increase of fatigue strength in a coated part as a result of the residual stress state.

As all residual stress state has the influence on the service characteristics of the coated parts, it is essential to know where the advantages of residual stresses actually occur and can successfully be used. The optimized use of residual stress states is a challenge to modern coatings engineering.

The following conclusions can be drawn:

a) residual stresses must be taken into account, if reliable coated parts are produced;

b) reliable methods for residual stress determination in coatings and coated parts are of great theoretical and practical importance;

c) great necessity for the information about the really existing residual stress states in the coated parts exists.

Taking into consideration the above-mentioned, this thesis is to improve the existing methods and to create new ones for determination of residual stresses in the coatings and coated parts dedicated. The residual stresses in some coatings are investigated as well.

The thesis is based upon the investigations carried out during the period 1962-1992 at the Estonian Agricultural University. The results of these investigations, published in the articles [1-25], have been taken as the basis of the thesis.

Out of 25 main publications on the subject of the thesis 12 articles have been published in the editions with international distribution and preliminary reviewed. In team-works [16, 18-25] the candidate has been the supervisor and the equal co-author as well. In the thesis [88] written under the supervision of the candidate, results of investigations published in papers [16, 18, 19, 22, 24, 25] have been partly used.

The main results of the thesis have been introduced at the following international and U.S.S.R. conferences and colloquiums:

1. Всесоюзная научно-техническая конференция "Остаточные напряжения и несущая способность деталей машин" (Харьков, 1969).
2. XIV научное совещание по тепловым напряжениям в элементах конструкций (Киев, 1977) [26].
3. The 6th International Conference "Experimental Stress Analysis" (Munich, 1978).
4. V и XIII Всесоюзные научно-технические конференции по конструкционной прочности двигателей (Куйбышев, 1978, Самара, 1991) [27,28,31].
5. II-V Kolloquien "Eigenspannungen und Oberflächenverfestigung "mit Iternationaler Beteiligung (Zwickau, 1979, 1982, 1985, 1989) [29].
6. The 9th International Conference on Experimental Mechanics (Copenhagen, 1990).
7. The European Conference "Residual Stresses" (Frankfurt a. M., 1992) [32,33].

The main part of the thesis consists of the introduction, the list of symbols, five chapters, the conclusions and the bibliography. Three examples of the application are placed in the appendix.

Background

The system of designations which is widely used classifies three different kinds of residual stresses [59]. From the engineering point of view the first kind or macro residual stresses are considered in the present thesis as of most interest.

Residual stresses cannot directly be determined. Distinct deformation parameter of a substrate or coating has always to be measured by which the value of residual stresses can be calculated. At the same time the difference has to be made between non-destructive and destructive measuring methods. In the non-destructive methods the deformation parameter is measured by applying the coating on the substrate, in the destructive methods by removing it from the substrate.

In the course of time various methods have been developed to determine residual stresses in the coatings.

E.J. Mills who published the article "On electrostriction" in 1877 [49] is generally considered as the first researcher of residual stresses in coatings (see, e.g. [58]). By his method the galvanic coating is deposited on the outer surface of a silvered vessel of a mercury thermometer and residual stresses are conventionally valued in scale units of the thermometer.

G.Stoney [57] is the first author of the quantitative method of determination of residual stresses in galvanic coatings. By his method the initial stress (the stress of superficial layer of coating) is calculated by the curvature measured after unilateral coating of a free straight strip substrate. At the same time the state of stress of coating and substrate is considered uniaxial and initial stress - constant. It is worth to mention that Stoney observed the coating process as consecutive application of parallel elementary layers, i.e. he used the model that is known as the model of continuous growth in layers.

In the article [56] the method of measuring the curvature of the straight strip substrate with unilateral coating was developed for the case of fixed ends. A. Brenner and S. Senderoff [39] have essentially improved this method. They have used the substrate with slipping ends and formed the theory which allows to determine the initial as well as the residual stresses and consider the difference of elastic module of coating and substrate.

In the candidate's article [81] the attention is paid to the fact that the strip substrate, width of which is usually considerably bigger than thickness, may not be treated as a beam. The theory of the method of measuring the curvature of a unilaterally coated plate substrate for the various fixing conditions of substrate edges is presented.

In the article [91] the error has arisen in the developed theory of the method of measuring the curvature of a straight strip substrate with unilateral coating because of the identification of the radii of gyration of a bimetal and homogeneous bar. The author has given up the constancy hypothesis of initial stress.

The idea of the method of measuring the deformation of a unilaterally coated straight strip substrate belongs to the authors of the article [93], who have used it for the determination of initial stresses in thin galvanic coatings supposing the existence of uniaxial state of stress.

The method of measuring the angular deflection of a helical warped strip substrate with unilateral coating is dealt for the first time in the article [38], where constant initial stress is calculated by modified Stoney's formula on the assumption that the helix angle of the substrate coil is negligible. In the monograph [92] the method is advanced considering biaxial state of stress and abandoning the hypothesis of constant initial stress. The obtained formula includes the error mentioned above in connection with the article [91].

In the article [87] the theory of the method of measuring the deflection of an unclosed ring strip substrate with slipping edges and unilateral coating on the assumption uniaxial state of stress is presented. The determination of initial stresses by measuring of longitudinal deformation of the round wire substrate is observed in the article [54] on assumption that the circumferential stress is constant and prestressing takes place by the force of gravity.

In the article [36] the determination of residual stresses by measuring of longitudinal deformation during the etching of outer surface (coating) of the thick boron fibre is considered on the assumption of uniaxial state of stress.

For determination of residual stresses in the thick non-homogeneous coating of non-homogeneous cylinder the destructive method, presented in the article [47], may be used. It is essential to mention that the identity hypothesis of deformation distribution of non-homogeneous and homogeneous cylinders decreases the precision of the method. In the monograph [77] the shortcoming of the observed non-destructive method is abandoning of the radial displacement corresponding to the longitudinal stresses.

In the case of equal elastic parameters of coating and substrate it is possible to determine the residual stresses in cylindrical and spherical coatings by deformation parameters measured by X-ray method on the free surface of the coating during the removal process [50, 41]. The method presented in the articles [52, 53] allows to determine the residual stresses in the coating of the hollow sphere in the same conditions by circumferential deformation measured on the inner surface of the substrate during the removal of coating.

Thermostresses and deformations in the straight and curvilinear substrate with unilateral coatings on assumption of uniaxial state of stress are treated in the article [39]. Methods, presented in the article [81] allow to determine the thermostresses and deformations in the multilayer coatings of the plate substrate.

The survey of the literature allowed to make the following conclusions at the different stages of the research:

1. There are no algorithms for the determination of residual stresses in the non-homogeneous parts with non--homogeneous coating.

2. Existing theory of methods for determination of initial stresses requires improving.

3. There are no methods for determination of initial stresses directed to the use of modern techniques primarily of strain gages.

4. Methods for determination of thermostresses and deformations in non-homogeneous coating of non-homogeneous

substrate require improving.

5. Several problems connected with determination of residual stresses in coatings need experimental investigation (authenticity of model of continuous growth in layers, stress distribution in the edge region of coated parts, etc.).

Outline of the thesis

Having looked through the literature the following problems, the solutions of which are given in the thesis are set up:

1. To compose the algorithms for the determination of residual stresses in the thick multi-layered coating of multi-layered rectangular bars, plates, cylinders and spheres.

2. To improve the theory of the methods for determination of initial stresses:

a) to consider substrate and coating non-homogeneous and the state of stress biaxial in the deformation parameters measuring method of the unilaterally coated strip or plate substrate;

b) to consider the state of stress biaxial and possibility to prestress the substrate with elastic element in longitudinal deformation measuring method of bilaterally coated straight strip substrate;

c) to desist the hypothesis of constant circumferential stress and consider the possibility to prestress the substrate with elastic element in the longitudinal deformation measuring method of the round wire substrate;

d) to deduce the formula for the calculation of initial stress for the method of measuring the displacement of the inner surface of thin-walled spherical substrate;

e) to consider the substrate as a short cylindrical shell in theory of the circumferential deformation measuring method of the thin-walled ring substrate;

f) to deduce the formula for the angular deflection measuring method of the helical warped strip substrate as a cylindrical shell with curvilineared edges.

3. To elaborate the method for the determination of initial stresses in the thick cylindrical galvanic coating

and the method for determination of initial stresses in tampon-galvanic coating directed to the use of strain gages.

4. To improve the method for the determination of thermostresses and deformations in unilaterally coated strip and plate substrate considering non-homogeneous coating and substrate and biaxial character of state of the stress. To compose the algorithms for determination of thermostresses and deformations in multi-layered cylinders and spheres with multi-layered coating.

5. To verify experimentally the residual stress distribution determined by the model of continuous growth in layers.

6. To determine the distribution of shear residual stresses in the edge region of unilaterally coated strip substrate and to check it up experimentally.

7. To verify experimentally deformation of strip or plate substrate at unilateral coating growth or removal.

8. To verify experimentally the theory of pure bending of a helical warped strip substrate with unilateral coating.

9. To investigate residual stresses in thick galvanic steel coatings used for rebuilding machine parts. To find a suitable function for description of initial stress distribution in galvanic coatings.

10. To compose the general algorithm for the determination of residual stresses in coated parts.

For solving enumerated problems the methods of linear theory of elasticity and the methods of technical theory of shells and plates were used. In the case of the strip and plate substrate the coating and substrate are considered continuously non-homogeneous, in case of cylindrical and spherical substrate - piecewise non-homogeneous.

In the experimental investigations strain gages, holographic interferometry and the photoelastic method were used.

LIST OF SYMBOLS

In the thesis following symbols are repeatedly used:

c – dimensionless parameter

E_m – modulus of elasticity of substrate (m=1) and coating (m=2)

$E'_m = E_m/(1-\mu_m)$ – elastic parameter of substrate (m=1) and coating (m=2)

e – distance of reduction surface from interface of coating and bar, plate or shell substrate

$f = 1 + \nu(4\zeta+6\zeta^2+4\zeta^3+\nu\zeta^4)$ – quartic polynomial

$f_1 = 1 + 3\zeta+3\nu\zeta^2+\nu\zeta^3$ – polynomial of the third degree

$f_2 = 1 + 2\zeta+\nu\zeta^2$ – polynomial of the second degree

h – variable thickness of coating

h_1 – thickness of the substrate

h_2 – thickness of the coating

$k = h_2/h_1$ – dimensionless thickness of coating

$k_o = r_o/r_1$ – dimensionless inner radius of cylindrical or spherical substrate

$k_2 = r_2/r_1$ – dimensionless outer radius of cylindrical or spherical coating

R – radius of moving surface (superficial layer) of cylindrical or spherical coating

r – radial coordinate

r_o – inner radius of cylindrical or spherical substrate

r_1 – outer radius of cylindrical or spherical substrate

r_2 – outer radius of cylindrical or spherical coating

α_m – coefficient of thermal expansion of substrate (m=1) and coating (m=2)

ΔT – temperature increment

ε – relative linear deformation on the free surface of bar, plate or shell substrate

$\zeta = h/h_1$ – dimensionless variable thickness of coating

η – z/h_1 – dimensionless coordinate

$\varkappa$ – curvature of free surface of bar or plate substrate

$\vartheta = E_2/E_1$ – ratio of moduli of elasticity of coating and substrate

μ_m – Poisson's ratio of substrate (m=1) and coating (m=2)

$\nu = E'_2/E'_1$ – ratio of elastic parameters of coating and substrate

$\xi = R/r_1$ – dimensionless radius of moving surface of cylindrical or spherical coating

$\rho = r/r_1$ – dimensionless radial coordinate

σ_{ijm} – residual stress components of the substrate (m=1) and coating (m=2) in rectangular coordinates (i, j=x, y,z)

σ_{im} – residual and thermal stress components of the substrate (m=1) and coating (m=2) in cylindrical (i=r, θ, z) or spherical (i=r, θ) coordinates

σ_m – residual stress in substrate (m=1) and in coating (m=2) in case of uniaxial or planar stress state with equal principal stresses

$\bar{\sigma}_2$ – initial stress

$\bar{\sigma}_2(0)$ – initial value of initial stress

σ^*_{ij2} – components of additional stress in rectangular coordinates (i, j=x, y, z)

σ^*_{i2} – components of additional stress in cylindrical (i = r, θ, z) and spherical (i = r, θ) coordinates

1. DETERMINATION OF RESIDUAL STRESSES IN COATED PARTS: GENERAL ALGORITHM

1.1. Model of continuous growth in layers. Initial stress

Formation of residual stresses in coating is directly connected with coating process. Since the precise mathematical interpretation of this process is complicated, it is necessary to give a growth model of coating.

The basis of the author's works is the model of continuous growth in layers, according to which the coating process is regarded as a continuous growth of the equidistant to substrate surface elementary layers. The general algorithm for determination of residual stresses based on this model is introduced in detail.

Let us consider the shell-like substrate (part) with the constant thickness h_1 one lateral surface of which is covered with coating (Fig.1.1). We take the principal lines of curvature of interface between the substrate and coating for coordinates x, y, i.e. in general x and y are curvilinears. Rectilinear coordinate z is taken along the normal of the interface towards the coating.

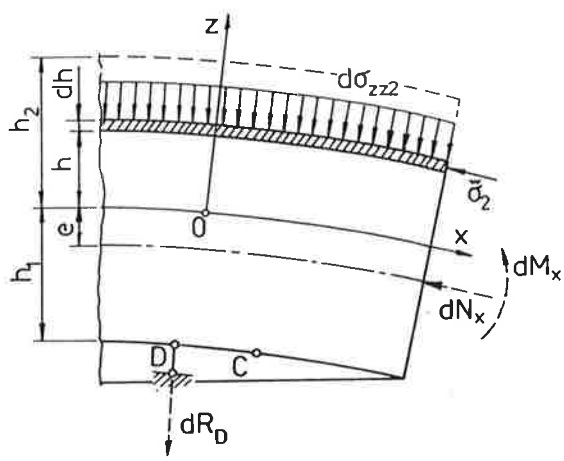

Fig.1.1 Coated substrate. The formation of the elementary layer dh of coating creates the loads $d\sigma_{zz2}$, dN_x, dM_x and reaction dR_D of redundant constraint

We assume the outer surface of the substrate is smooth with positive Gaussian curvature and the edges of the substrate congruent with the principal line of curvature of the interface. Supposing the substrate is sufficiently fixed, i.e. the degree of freedom of the substrate as a rigid body is zero or negative.

According to the model of continuous growth the coating is formed as the result of successive addition of elementary layers equidistant from the substrate surface. Let us denote the variable coating thickness by **h**. We investigate, presuming the surface z=**h**+d**h** is free, i. e. there are no stresses on the surface and displacements are not prevented.

Physico-chemical processes in the superficial layer cause the volume change [72], which in the case of isotropic material characterized by deformations

$$\bar{\varepsilon}^{o}_{ij2} = \bar{\varepsilon}^{o}_{2}\delta_{ij} \quad (i,\ j = x,\ y,\ z), \tag{1.1}$$

where $\bar{\varepsilon}^{o}_{2} = \bar{\varepsilon}^{o}_{2}(\mathbf{h})$ is the initial deformation and δ_{ij} is Kronecker's symbol.

Assuming the superficial layer d**h** being in the ideal mechanical contact with the coating we can come to the conclusion that from the deformations (1.1) only the defortion $\bar{\varepsilon}^{o}_{zz2}$ in normal direction can be freely realized. The deformations $\bar{\varepsilon}^{o}_{xx2}$ and $\bar{\varepsilon}^{o}_{yy2}$ of some superficial layer element in tangential plane are prevented because the processes causing the volume change occur also in the adjacent elements. Hence the biaxial state of stress arises in the superficial layer.

Depending on the intensity of the initial deformation $\bar{\varepsilon}^{o}_{2}$ the material of superficial layer may deform either an elastic or a plastic. Since the results obtained by the mechanical methods for the determination of residual stresses do not depend on the mechanical state of superficial layer the treatment is limited to elastic state. (More general elastoplastic treatment is given in the articles [1,84]).

According to the addition principle it is possible to calculate the total deformations in the superficial layer as the sum of elastic deformations $\bar{\varepsilon}^{e}_{ij2}$ and initial deformations (1.1):

$$\bar{\varepsilon}_{ij2} = \bar{\varepsilon}^{e}_{ij2} + \bar{\varepsilon}^{o}_{2}\delta_{ij}. \tag{1.2}$$

Expressing the elastic deformations in the formulas (1.2) by stresses using Hooke's equations, we obtain

$$\bar{\varepsilon}_{ij2} = (1/E_2)[(1+\mu_2)\bar{\sigma}_{ij2} - \mu_2\bar{\sigma}_{\Sigma 2}\delta_{ij}] + \bar{\varepsilon}^{o}_{2}\delta_{ij}, \qquad (1.3)$$

where $\bar{\sigma}_{ij2} = \bar{\sigma}_{ij2}(\mathbf{h})$ are stresses and $\bar{\sigma}_{\Sigma 2}$ is the sum of normal stresses in the superficial layer.

Bearing in mind that after formation the superficial layer forms the whole with coating and all its elements, five of the six total deformations may be taken equal to zero:

$$\bar{\varepsilon}_{xx2} = \bar{\varepsilon}_{yy2} = \bar{\varepsilon}_{xy2} = \bar{\varepsilon}_{yx2} = \bar{\varepsilon}_{zx2} = 0. \qquad (1.4)$$

Taking into account condition $\bar{\sigma}_{zz2} = 0$, we obtain from the expressions (1.3) the system of equations whence we find the stresses in the superficial layer:

$$\bar{\sigma}_{xx2} = \bar{\sigma}_{yy2} = \bar{\sigma}_{2} = -E_2\bar{\varepsilon}^{o}_{2}/(1-\mu_2),$$
$$\bar{\sigma}_{xy2} = \bar{\sigma}_{yz2} = \bar{\sigma}_{zx2} = 0. \qquad (1.5)$$

The elastic deformations in the superficial layer by the initial deformation can be expressed with the help of the equations (1.2) and the conditions (1.4). We obtain

$$\bar{\varepsilon}^{e}_{xx2} = \bar{\varepsilon}^{e}_{yy2} = \bar{\varepsilon}^{e}_{2} = -\bar{\varepsilon}^{o}_{2}. \qquad (1.6)$$

Thus, biaxial state of stress with equal principal stresses arises in the superficial layer. The stress $\bar{\sigma}_2 = \bar{\sigma}_2(\mathbf{h})$ in the superficial layer is called initial stress. It is essential to note that the stress does not depend on rigidity and fixing conditions of the coated substrate.

1.2. General algorithm for determination of residual stresses

Let us observe the formation of stress state in the internal layer $z<\mathbf{h}$ of the coating. Initial stress (1.5) in the layer dz arise immediately after its formation ($\mathbf{h}=z$). The additional stresses $\sigma^{*}_{ij2}(X)$ will be added to the initial stresses during the coating growth in the interval $z < \mathbf{h} \le \mathbf{h}_2$. For brevity X denotes here and below the coordinates of point (x, y, z).

Thus, residual stresses in the coating are expressed as the sum of initial and additional stresses:

$$\sigma_{ij2} = \bar{\sigma}_2(z)\delta_{ij} + \sigma^{*}_{ij2}(X) \quad (\delta_{zz} = 0). \qquad (1.7)$$

The additional stresses are calculated by integrating the elementary additional stresses $d\sigma^*_{ij2}(X,\mathbf{h})$ arising in the layer dz by the formation of the superficial layer $d\mathbf{h}$:

$$\sigma^*_{ij2} = \int_z^{h_2} d\sigma^*_{ij2}(X,\mathbf{h}). \tag{1.8}$$

The residual stresses, deformations and displacements in the substrate $(-\mathbf{h}_1 \leq z \leq 0)$ are expressed:

$$\begin{bmatrix} \sigma_{ij1} \\ \varepsilon_{ij1} \\ u_{i1} \end{bmatrix} = \int_0^{h_2} \begin{bmatrix} d\sigma_{ij1}(X,\mathbf{h}) \\ d\varepsilon_{ij1}(X,\mathbf{h}) \\ du_{i1}(X,\mathbf{h}) \end{bmatrix}, \tag{1.9}$$

where $d\sigma_{ij1}$, $d\varepsilon_{ij1}$ and du_{i1} are elementary stresses, deformations and displacements, arising in the substrate by the formation of the superficial layer $d\mathbf{h}$.

We explicate initial stress in the equations (1.7) and (1.8). For this it is possible to use the solution of corresponding thermoelastic problem assuming, according to the formula (1.5) thermodeformation

$$\varepsilon_t = \bar{\varepsilon}^0_2 H(z-\mathbf{h}) = -[(1-\mu_2)\bar{\sigma}_2/E_2]H(z-\mathbf{h}) \tag{1.10}$$

in the substrate and coating is present. In the latter formula $H(z-\mathbf{h})$ is Heaviside's function.

This method is used in the articles [82-84]. For the same purpose the mechanical effect caused by the formation of the superficial layer $d\mathbf{h}$ could be replaced by the surface and edge loads acting on the coating.

To determine the surface load the equilibrium equation $\Sigma Z = 0$ for an element of superficial layer $d\mathbf{h}$ is composed. Hence we can find

$$d\sigma_{zz2} = \mp \bar{\sigma}_2(R_x^{-1} + R_y^{-1})d\mathbf{h}, \tag{1.11}$$

where R_x and R_y are the principal radii of curvature of the superficial layer. The sign (−) is valid for the coating on the convex and (+) on the concave side of the substrate.

If the coating has a free edge (Fig.1.1), the homogeneous state of stress (1.5) at the edge of superficial layer is possible only on the condition that the edge is loaded with the initial stress $\bar{\sigma}_2$. As in practice this load does not exist, an opposite-directed stress $-\bar{\sigma}_2$ which relieves the edge from the stress is applied to the edge of coating. In the case of shell-like or plate-like substrate (Fig.1.1) it is proper to reduce the stress $-\bar{\sigma}_2$ to an edge load which consists of edge force and edge moment:

$$dN_x = -\bar{\sigma}_2 d\mathbf{h}, \quad dM_x = \bar{\sigma}_2 (e+\mathbf{h}) d\mathbf{h}, \qquad (1.12)$$

where $e = e(\mathbf{h})$ is the distance from the substrate and coating interface to the reduction surface.

In the case of the isotropic substrate and coating with elastic parameters variable through the thickness the elastic equations of the theory of shells simplify considerably, if we determine the position of reduction surface from the condition [65]:

$$\int_{-h_1}^{h} [E/(1-\mu)] z dz = 0, \qquad (1.13)$$

where $E = E(z)$ and $\mu = \mu(z)$ are the modulus of elasticity and Poisson's ratio. If the elastic parameters of the substrate and coating are constant and equal, from the condition (1.13) we find $e = (\mathbf{h}_1 + \mathbf{h}_2)/2$, i.e. the reduction surface is the middle surface.

We note that depending on the fixing conditions of the substrate the edge load (1.12) may be either partly or fully balanced by reactions of constraints. For example, in the case of slipping edge the edge moment has been balanced and in the case of fixed edge both the edge moment and the edge force are balanced.

Use of the edge load as forces and moments reduced to the edge of the reduction surface means to satisfy the statical edge conditions in Saint-Venant's meaning. Therefore the obtained solution may be considered quite exact outside the edge region, the size of which usually does not exceed the summarized thickness of substrate and coating.

Solving the thermoelastic problem for the certain substrate with coating by thermodeformation (1.10) or the problem of theory of elasticity by loads (1.12) we can get the elementary stresses, deformations and displacements through initial stress within the framework of linear elasticity as follows:

$$\begin{bmatrix} d\sigma^{*}_{ij2} \\ d\sigma_{ij1} \\ d\varepsilon_{ij1} \\ du_{i1} \end{bmatrix} = \bar{\sigma}_2(h) \begin{bmatrix} f^{-1}_{ij2}(X,h) \\ f^{-1}_{ij1}(X,h) \\ g^{-1}_{ij1}(X,h) \\ p^{-1}_{i1}(X,h) \end{bmatrix} dh, \qquad (1.14)$$

where f_{ij2}, f_{ij1}, g_{ij1} and p_{i1} are the functions characterizing the rigidity of coated substrate which depend on the shape, dimensions and fixing conditions of the substrate and the elastic parameters of the substrate and coating.

Substituting the elementary stresses, deformations and displacements from (1.14) in the expression (1.8) and (1.9) we will have the following general formulas for calculation of the stresses, deformations and displacements in the coating and substrate:

$$\sigma^{*}_{ij2} = \int_{z}^{h_2} \bar{\sigma}_2(\mathbf{h}) f^{-1}_{ij2}(X,\mathbf{h}) d\mathbf{h}, \qquad (1.15)$$

$$\begin{bmatrix} \sigma_{ij1} \\ \varepsilon_{ij1} \\ u_{i1} \end{bmatrix} = \int_{0}^{h_2} \bar{\sigma}_2(h) \begin{bmatrix} f^{-1}_{ij1}(X,h) \\ g^{-1}_{ij1}(X,h) \\ p^{-1}_{i1}(X,h) \end{bmatrix} dh. \qquad (1.16)$$

The formulas (1.7), (1.15) and (1.16) allow to calculate the residual stresses in coating and substrate, and the residual deformations and displacements in the substrate, if the initial stress $\bar{\sigma}_2(\mathbf{h})$ is known. Since in the surface physics of materials there are no theoretical methods for the determination of initial stress the following experimental methods are used:

1.In case of deformation methods the initial stress is determined by the deformation parameters measured either on the free surface of the substrate ($z = -\mathbf{h}_1$) or on the moving surface of the coating ($z = \mathbf{h}$). For example, measuring the deformation $\varepsilon_{xx1C}(\mathbf{h})$ with strain gage or linear displacement $u_{z1c}(\mathbf{h})$ with the displacement gage at some point C of the free surface of substrate (Fig.1.1), we can calculate the initial stress by the formulas

$$\bar{\sigma}_2 = \begin{cases} g_{xx1C}(\mathbf{h})\ d\varepsilon_{xx1C}(\mathbf{h})/d\mathbf{h} \\ p_{z1C}(\mathbf{h})\ du_{z1C}(\mathbf{h})/d\mathbf{h} \end{cases} \qquad (1.17)$$

resulting from the expressions (1.14).

On the moving surface of the coating it is possible to determine the elastic deformation (1.6) corresponding to the initial stress by chrystallographic parameters of deformation measured by X-ray method.

2. In the case of force methods the initial stress is determined by reactions of redundant constraints preventing the deformation of substrate during coating process. For example, by measuring the reaction $R_D(\mathbf{h})$ of constraint D (Fig.1.1) the initial stress is calculated by formula

$$\bar{\sigma}_2 = q_D(\mathbf{h})dR_D(\mathbf{h})/d\mathbf{h}, \qquad (1.18)$$

where $q_D(\mathbf{h})$ is the function characterizing the rigidity of the coated substrate.

It is possible to determine the initial stress by a variant of deformation method when the deformation parameter caused by the removal of a redundant constraint is measured. For example, the displacement $u_{z1CD}(\mathbf{h})$ of the substrate point C after the removal of the redundant constraint D is measured, then according to Hooke's law

$$u_{z1CD}(\mathbf{h}) = r^{-1}_{CD}(\mathbf{h})R_D(\mathbf{h}),$$

where $r_{CD}(\mathbf{h})$ is the function characterizing the rigidity of coated substrate without redundant constraint.

Substituting the reaction $dR_D(\mathbf{h})$ found from the formula (1.18) by integrating into the latter expression we will obtain the integral equation for determination of initial stress

$$u_{z1CD}(\mathbf{h}) = r^{-1}_{CD}(\mathbf{h})\int_0^h \bar{\sigma}_2(\mathbf{h})q^{-1}_D(\mathbf{h})d\mathbf{h}. \qquad (1.19)$$

Usually the redundant constraints for constraining the deformation of substrate are applied to the substrate edges. The suitable force methods and corresponding deformation methods for the bar-like and plate-like substrate with slipping and fixed edges have been dealt with in the paper [6]. The theory of deformation methods of the bar and plate-like substrate during the coating process with fixed edges is presented in the articles [8,15].

The formulas (1.7) and (1.15)-(1.19) form the general algorithm for determination of residual stresses, deformations and displacements in coated parts by the deformation parameters measured on the free surface of the substrate or on the moving surface of the coating. We demonstrate that the presented algorithm is valid also in the case if deformation parameters being measured during the process of coating removal, i.e. the destructive method is used.

We assume that the coating removal process (electrolytic or chemical etching, polishing, laser treatment, etc.) is quite precisely to be described by the model of continuous decrease of coating in layers, according to which the removal process of coating is regarded as a continuous removal of the equidistant to substrate surface elementary layers. As shown above, the formation of superficial layer dh of coating is statically equivalent with the application of the surface and edge loads (1.11), (1.12). The removal of the same superficial layer is statically equivalent to releasing from above-mentioned loads, i.e. with the application of the negative loads (1.11) and (1.12). If the technology of removal does not influence the initial stress in the superficial layer, the removal of the superficial layer causes elementary stresses, deformations and displacements in the substrate and coating different from the corresponding quantity at the forming of the superficial layer only in sign.

Thus, the removal and growth process of the coating are invertible on certain conditions, i.e. the initial stress value does not depend on whether the deformation parameters used for its determination were measured either on removal or growth process. It follows that the algorithm (1.7), (1.15)-(1.19) is universal for non-destructive and destructive methods.

It is supposed in the present algorithm that the deformation parameters for the determination of initial stress are measured during coating process of the part in which the residual stresses are determined. In practice such measurements are usually technically complicated or are altogether impossible. The point is the substrate deformation parameters are often too little for the common gages or there is no free surface of the substrate for measuring the deformations and displacements. To overcome these difficulties the author has developed the method [1, 2, 83, 84] which in the special literature is called computational-experimental method (see, e.g. [70,79]). The algorithm of this method formally coincides with the abovementioned universal algorithm. The essential difference is that for determination of initial stress the coating is applied not on the part, but on the thin-walled substrate of the same material and also, if necessary geometrically similar thin substrate. Since the rigidity of substrate is considerably smaller than rigidity of part, it is possible to measure the deformation parameters by the help of common instrument quite exactly.

In some cases the use of a geometrically similar thin-walled substrate may be the only way for investigation of residual stresses in the coated massive part. For example, we may take a coated sphere when the substrate has no free surface at all and thus measuring of the substrate deformation by common techniques is impossible. Nevertheless, the residual stress can be determined by applying the coating to the thin-walled spherical substrate and measuring the radial displacement of inner surface by the principle of liquid thermometer [49 ,84].

It is expedient to distinguish thick and thin coatings. The importance of additional stresses in residual stresses may be taken as the basis of the classification. If we have thicker coating, additional stresses are of greater importance. If we ignore the additional stresses up to 5%, we may consider the coating of momentless substrate thin on the condition $\nu k < 1/20$ and coating of substrate with the moment on the condition $\nu k \leq 1/80$ [1].

Most of the coatings used in practice are thin. Residual stresses in a part with thin coating are ignorable, but in the coating equal to initial stress.

The algorithm (1.7), (1.15)-(1.19) is based on the model of continuous growth of coating, which enables to reduce the mechanical effect of the elementary layer applying to elementary loads (1.11), (1.12) using the initial stress concept. Such a treatment in differential form used in the most of the author's works (see, e.g. [4, 17, 25]), may be called a differential approach. Analogically to the destructive methods [69], the integral approach is possible (see, e.g. [3, 9, 10, 13]).

For the elimination of edge region by integral approach the modified Birger's scheme (Fig.1.2) can be used. Unlike his destructive method, based on the principle of the application of opposite-direction residual stresses [69] in the case of coating growth, the residual stresses must be applied to the original direction.

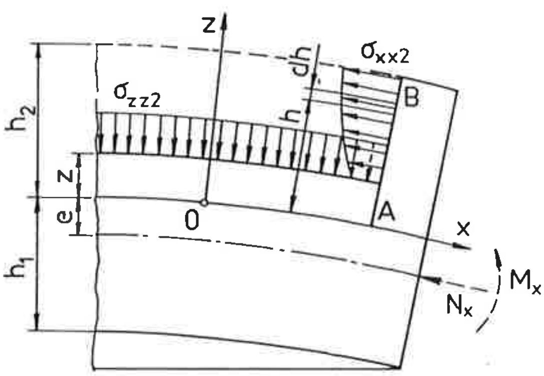

Fig.1.2. Modified Birger's scheme for the elimination of edge effect. The static effect of formation of the finite thickness layer h_2-z of the coating: loads σ_{zz2}, N_x, M_x

The differential and integral approach mainly differ in that the model of continuous growth of coating and initial stress in explicit form are not used in the latter case. If in the differential approach the treatment was based on the consideration of forming of elementary superficial layer, mechanical effects of which were elementary loads (1.11), (1.12), then in the integral approach the basis for the treatment was the formation of finite thickness layer h_2-z and the surface load $\sigma_{zz2}(X,z)$ and edge loads

$$N_x = \int_z^{h_2} \sigma_{xx2}(X,h)dh, \quad M_x = \int_z^{h_2} \sigma_{xx2}(X,h)[e(z) + h]dh \qquad (1.20)$$

statically equivalent to it. Here $\sigma_{zz2}(X,z)$ is the residual stress affecting on the surface $h = z$ and $\sigma_{xx2}(X,h)$ is the residual stress affecting on the surface AB in the Birger's scheme (Fig.1.2).

By the differential approach we assumed that the initial stress depends only on the variable thickness h of coating and as result we get the algorithm which helps to determine the initial stress and residual stress by deformation parameters measured only at one point of the free surface of substrate or moving surface of coating. In order to get the algorithm with the same possibilities by integral approach we have to assume the residual stresses dependent only on the coordinate z. We know, such an assumption constrains the range of the problems solved by the integral approach because of the circumstance that unlike the initial stresses, the residual stresses depend on geometric and mechanical parameters of the substrate.

For introducing the integral approach let us observe the determination of residual stresses by growth of freely deforming plate with arbitrary contour [3]. On assumption that out of the edge zone $\sigma_{zz2}(z) = 0$, $\sigma_{xx2}(z)=\sigma_{yy2}(z)=\sigma_2(z)$ the problem is reduced to the problem of compression and bending of the plate with the edge load

$$N = \int_z^{h_2} \sigma_2(h)dh, \quad M = \int_z^{h_2} \sigma_2(h)[(h_1-z)/2 + h]dh.$$

Solving this problem in the framework of the technical theory of plates and supposing that $E_1 = E_2 = E$ and $\mu_1 = \mu_2 = \mu$, we get the equation

$$2 \int_z^{h_2} \sigma_2(h)(h_1-2z + 3h)dh = E'(h_1+z)^2\Delta\varepsilon(z), \qquad (1.21)$$

where $\Delta\varepsilon(z) = \varepsilon(h_2)-\varepsilon(z)$ is the increment of deformation measured on the free surface of substrate during coating growth at the interval (z, h_2).

The equation (1.21) is Volterra's first kind integral equation from which, by means of the reduction to differential equation, is found:

$$\sigma_2=-E'\left[\frac{h_1+z}{2}\,\frac{d\Delta\varepsilon(z)}{dz}+2\Delta\varepsilon(z)-3(h_1+z)\int_z^{h_2}\frac{\Delta\varepsilon(h)}{(h_1+h)^2}\,dh\right]. \qquad (1.22)$$

It is easy to be convinced that the first member of the latter formula determines the initial stress in the superficial layer. Indeed, the passage to the limit $h_2\to z$ gives

$$\bar{\sigma}_2 = -(E'/2)(h_1+z)d\Delta\varepsilon(z)/dz. \qquad (1.23)$$

Thus, in the case of the integral approach the initial stress is included in the final result in the implicit form. If the function $\Delta\varepsilon(z)$ is to be found by integrating the differential equation (1.23) and then to express $\Delta\varepsilon(h)$ in the second and the third member of the equation (1.22) by initial stress, we get

$$\sigma_2 = \bar{\sigma}_2(z) + 2\int_z^{h_2}\frac{\bar{\sigma}_2(h)}{h_1+h}\,dh - 6(h_1+z)\int_z^{h_2}\frac{\bar{\sigma}_2(h)}{(h_1+h)^2}\,dh. \qquad (1.24)$$

We get the same result by the differential approach while the solution process is easier, as there is no need to compose and solve the integral equation. Still we cannot conclude that the differential approach always permits to avoid the integral equation. If coating (e.g. on technological consideration) has been applied on the substrate with redundant constraint and the stresses are determined by the deformation parameters generated on releasing from redundant constraints, the determination of the initial stress reduces the solving of equations similar to the integral equation (1.19). In some cases, e.g. by applying a coating to the all outer surface of the spherical substrate, the moving surface is closed and the edge load does not exist at all. In that case the formation of some coating layer of finite thickness h_2-z is statically equivalent to the surface load $\sigma_{zz2}(z)$ applying and the integral and differential approaches are practically of the same effectiveness.

As shown above, the algorithm of the differential approach is valid on certain assumption regardless, if the deformation parameters on the substrate or coating were measured at the process coating growth or removal. Naturally, the above given is valid for the algorithm of integral approach as well. In this connection we note, that in the integral approach algorithm the increment of deformation parameters of substrate is used as experimental information. For example, in the formulas (1.21)-(1.23) it is represented by the deformation increment $\Delta\varepsilon(z) = \varepsilon(\mathbf{h}_2)-\varepsilon(z)$. If, for the determination of residual stresses the coating is removed and during the growing process the deformation has not been measured, $\varepsilon(\mathbf{h}_2) = 0$ is taken, i.e. $\Delta\varepsilon(z) = -\varepsilon(z)$. The algorithm of differential approach does not need such a correction since the experimental information is used in the form of deformation parameter derivation.

We add to the integral approach its principles are formulated in the work [80] as general principle of non-destructive mechanical methods.

We have found residual stresses in the substrate (1.16) satisfy usual compatibility conditions of linear elasticity. If to compose the compatibility conditions for coating, taking into consideration the initial deformation (1.1) and paying no attention to the layer growth process of coating, we assume the physico-chemical processes generate the initial stress which appear after growth only, then the residual stresses calculated by the formulas (1.7) and (1.15) do not satisfy these conditions. For example, in the above considered case of free plate growth the compatibility condition for the coating is given in the form: $d^2\sigma^*_2(z)/dz^2 = 0$. Substituting here the additional stress (the second and the third member of the formula (1.24)), the result will be

$$d^2\sigma^*_2(z)/dz^2 = [4/(\mathbf{h}_1+ z)]d\bar{\sigma}_2(z)/dz + 2\bar{\sigma}_2(z)/(\mathbf{h}_1+ z)^2,$$

i.e. compatibility condition is not satisfied.

The author has obtained the analogical result in the case of cylinder and sphere coating [83, 84]. The fact that residual stresses in coating do not satisfy usual compatibility condition of a body with stationary configuration is explicable with the specific feature of the coating formation, which consists in discontinuity of deformation and stress distribution on the interface of coating and superficial layer dh.

1.3. Treatment of experimental information

According to both above-mentioned algorithm variants the residual stresses are calculated by deformation parameters experimentally determined. Since the experimental data always contain errors to gain authentic results they have to be treated mathematically [48]. The measurement errors are to be carefully considered as the presented algorithm requires the differentiation of experimental data, which, as known, is ill-posed problem (small measurement errors of initial data may cause big errors in their derivative values).

In the author's works the following methods receiving the authentic results of the derivation of experimental data have been used:

1. Previous smoothing of the experimental data with cubic splines [17, 18, 29] or polynomial, formed by the use of the least-squares method [13].

2. Calculating the derivative through solving the integral equation by regularization method [24, 94].

3. Approximation of experimental data with semiempiric formulas, gained on the assumption that the dependence of initial stress on the coating thickness is describable by linear fractional function

$$\bar{\sigma}_2 = \bar{\sigma}_2(0)(h_1 + \nu h)/(h_1 + c\nu h), \qquad (1.25)$$

where $\bar{\sigma}_2(0)$ is initial value of initial stress and c is dimensionless parameter. If c = 1 then $\bar{\sigma}_2(h) = \bar{\sigma}_2(0)$ =const, if c < 1 the initial stress increases and if c > 1 decreases monotonically.

By using the formula (1.25) we obtain from the first expression (1.17) the following semiempiric formula for approximation of deformation ε_{xx1C}:

$$\varepsilon_{xx1C} = \bar{\sigma}_2(0) \int_0^h g^{-1}_{xx1C}(h)[(h_1 + \nu h)/(h_1 + c\nu h)]dh. \qquad (1.26)$$

For determination of parameters $\bar{\sigma}_2(0)$ and c in the author's works [2, 5, 85] there has been used the method of equal averages [76], in the article [3] - iterative method [76] and in works [16,22] - deforming polyhedron method [46] with least-squares method where the integral was calculated by Simpson's rule.

Describing the initial stress distribution by the formula (1.25) there are the following advantages:

a) the initial stress distribution in coating is determined by two parameters, while parameter $\bar{\sigma}_2(0)$ determines ordinarily the greatest value of initial stress in coating;

b) it is possible to prognosticate initial stress distribution in coating, the thickness of which differs from the thickness of experimental coating;

c) if the parameter c is known for the certain class of coating, the value of deformation parameter measured at coating of some simple substrate, e.g. strip-like substrate is enough for the express check of the residual stress.

The linear fractional distribution of initial stress cannot be universal. Experimental researches have shown that the formula (1.25) enables effectively to describe the initial stress distribution during the galvanic deposition in a bath [3, 5, 22, 27, 30, 85] and by brushes [16].

2. DETERMINATION OF RESIDUAL STRESSES IN BARS, PLATES, CYLINDERS AND SPHERES WITH THICK COATING

In this chapter a short survey of the author's works is given, which deal with determination of residual stresses in thick coatings. Due to the limited size of the thesis a detailed treatment of general algorithm composed for the case of multilayer coatings and substrate has been omitted and the computational formulas for determination of residual stresses and deformations for more frequently occurring special case of coating technology have been presented when substrate and coating are homogeneous, i.e. their elastic parameters are constant. For the sake of unification of presentation the results have been given in the form, obtained by the differential approach.

2.1. Free plates with unilateral coating

In the article [3] using the integral approach the formula is derived for the residual stresses in unilateral coating of rectangular bar or arbitrary contoured plate by deformation, measured on the free surface of substrate, on the supposition that elastic parameters of coatings and substrate are constant and equal. Semiempirical formula for approximation of experimental information is obtained from the linear-fractional expression (1.25).

In the article [10] the same problem is solved in more general form when the elastic constants of the coating and the substrate are different. At the same time Volterra's first kind integral equation, typical to the integral approach, is analytically solved by the reduction to differential equation and numerically using the Newton-Cotes' quadrature formulas.

In the report [85] using the differential approach on the ground of the hypothesis of technical theory of plates formulas are derived for the calculation of residual stresses in the unilateral coating of a rectangular bar or a plate by the deformation $\varepsilon(\zeta)$ or curvature $\varkappa(\zeta)$ of the free surface of a substrate on the supposition that the elastic parameters of the coating and the substrate change arbitrarily in the direction of thickness. From the formulas of the report [85] the next expressions follow for the special case of constant elastic parameters:

$$\bar{\sigma}_2 = \begin{cases} (E_1'/2)[f(\zeta)/f_1(\zeta)]d\varepsilon(\zeta)/d\zeta, & (2.1) \\ (E_1'h_1/6)[f(\zeta)/f_2(\zeta)]d\varkappa(\zeta)/d\zeta, & (2.2) \end{cases}$$

$$\sigma_2^* = 2\nu[P_1(\eta)-3(1+\eta)P_2(\eta)], \qquad (2.3)$$

$$\sigma_1 = 2P_1(0)-6(1+\eta)P_2(0). \qquad (2.4)$$

In these formulas

$$P_j(\eta) = \int_\eta^k [f_j(\zeta)/f(\zeta)]\bar{\sigma}_2(\zeta)d\zeta \qquad (j=1,\ 2).$$

2.2. Circular hollow cylinders with outer coating

The articles [83, 11, 13, 17] deal with the determination of residual stresses in the coated long circular cylinders. In the article [83] proceeding from the solution of thermoelastic problem of a hollow layered cylinder the algorithm has been obtained by the differential approach for determination of residual stresses in the coated cylinders on the supposition that the elastic parameters of coating and substrate are constant. The analogical, at the same time considering the difference between circumferential and longitudinal initial stress, algorithm obtained by integral approach has been presented in the article [11].

In the paper [13] there is the algorithm for determination of residual stresses in the thick coating of cylinders in the case of coating and substrate being non-homogeneous composed by the integral approach. The algorithm is based on the replacement of non-homogeneous coating and substrate with piecewise homogeneous layered cylinder and application of Lamé's formulas for the homogeneous layers, while functions, included in the formulas are expressed recurrently by the deformation measured on the free surface of the substrate using the continuity conditions of radial stresses and circumferential deformatioans on the interface of layers. This method succeeds in avoiding the solving procedure of linear system equations included in the known algorithms [83, 42, 43].

Let us present the algorithm given in the articles [11, 13] for the special case when the coating and the substrate are homogeneous and circumferential and longitudinal initial stress equals ($\bar\sigma_{xx2} = \bar\sigma_{yy2} = \bar\sigma_2$):

$$\bar\sigma_2 = \begin{cases} -[E_1/2(1-\mu_2)][q(\xi)/p_o(\xi)\xi]d\varepsilon_\theta(\xi)/d\xi, & (2.5) \\ -[E_1/2(1-\mu_2)][q(\xi)/p_2(\xi)\xi]d\varepsilon_z(\xi)/d\xi, & (2.6) \end{cases}$$

$$(\sigma^*_{r2}, \sigma^*_{\theta 2}) = -Q_4(\rho) \mp Q_6(\rho)/\rho^2, \qquad \sigma^*_{z2} = -2Q_8(\rho), \tag{2.7}$$

$$(\sigma_{r1}, \sigma_{\theta 1}) = -2(1-\mu_2)(1 \mp k_o^2/\rho^2)Q_{10}(1), \quad \sigma_{z1} = -2(1-\mu_2)Q_{12}(1). \tag{2.8}$$

In the formulas (2.5)-(2.8)

$$q(\xi) = a_o\xi^4 + a_1\xi^2 + a_2,$$

$$p_j(\xi) = b_j\xi^2 + b_{j+1} \qquad (j= 0, 2,... 12),$$

$$Q_j(\rho) = \int_\rho^{k_2} [p_j(\xi)\xi/q(\xi)]\bar{\sigma}_2(\xi)d\xi \qquad (j=4, 6,... 12),$$

$$a_o= \vartheta[2(1-\mu_1)\vartheta+(\lambda-\vartheta)(1-k_o^2)],$$

$$a_1= [2(1-\mu_2+\vartheta-\mu_1\lambda)\vartheta+(\lambda-\vartheta)(1-k_o^2)](1-k_o^2)-4(1-\mu_1)\vartheta^2,$$

$$a_2= \{[2(1-\mu_2)\lambda+\vartheta-\lambda](1-k_o^2)-\vartheta[2(1-\mu_1)\lambda+2(1-\mu_2)+\vartheta-\lambda]\}(1-k_o^2)+$$
$$+2(1-\mu_1)\vartheta^2,$$

$$b_o= \mu_1(\vartheta-\lambda)(1-k_o^2)+2(1-\mu_1)\vartheta, \quad b_1= [2\lambda-\mu_1(\vartheta+\lambda)](1-k_o^2)-2(1-\mu_1)\vartheta,$$

$$b_2= (\lambda-\vartheta)(1-k_o^2)+2(1-\mu_1)\vartheta, \qquad b_3= [(1-2\mu_1)\lambda+\vartheta](1-k_o^2)-2(1-\mu_1)\vartheta,$$

$$b_4= a_o, \qquad b_5=[2(1-\mu_1)\vartheta\lambda+(\lambda-\vartheta)(1-\vartheta-k_o^2)](1-k_o^2)-2(1-\mu_1)\vartheta^2,$$

$$b_6= \{[2(1-\mu_2)+\vartheta-\lambda](1-k_o^2)-2(1-\mu_1)\vartheta\}\vartheta, \qquad b_7= a_2, \qquad b_8= a_o,$$

$$b_9= \{\vartheta[(1-2\mu_1)\lambda+\vartheta]+\mu_2(\lambda-\vartheta)(1-k_o^2)\}(1-k_o^2)-2(1-\mu_1)\vartheta^2,$$

$$b_{10}= \vartheta, \qquad b_{11}= (1-k_o^2)\lambda-\vartheta, \qquad b_{12}= (\lambda-\vartheta)(1-k_o^2)+2\vartheta,$$

$$b_{13}= (\lambda+\vartheta)(1-k_o^2)-2\vartheta,$$

where

$$\lambda = (1+\mu_2)/(1+\mu_1).$$

The formulas (2.5)-(2.7) allow to determine the residual stresses in the external coating of a hollow cylinder by circumferential deformation $\varepsilon_\theta(\xi)$ or longitudinal deformation $\varepsilon_z(\xi)$ measured on the inner surface of the substrate. If the initial stress $\bar{\sigma}_2(\xi)$ is determined by X-ray method or by the deformation of a thin-walled substrate, the residual stresses in the coating and the substrate may be calculated by the formulas (2.7) and (2.8).

We note, in the case of equal elastic constants of the coating and the substrate ($E_1= E_2= E$, $\mu_1= \mu_2= \mu$) the formulas which in the articles [50, 41] are derived specially for the X-ray method follow from the expressions (2.7) and (2.8).

In the case of a solid substrate we have to take $k_o=0$ in the formulas (2.5)-(2.8). The adequate expressions have also been obtained by differential approach in the article [17], where they are used for the determination of residual stresses in fibres by longitudinal deformation of core measured during the removal of the outer layer (coating). In comparision with the methods recommended in the articles [36, 95] according to which the state of stress of coated fibres is considered uniaxial, the developed method makes it possible to determine all the three principal stresses of residual stress state (see Appendix, Example A.1).

Considering the coated thin-walled tubular substrate $(h_1, h_2) << r_1$ the basic formulas (2.5)-(2.8) could be simplified

$$\bar{\sigma}_2 = -E_1'(1+\nu\zeta)d\varepsilon(\zeta)/d\zeta, \tag{2.9}$$

$$\sigma_{r2} \approx 0, \quad \sigma^*_{\theta 2} = \sigma^*_{z2} = \sigma^*_2 = -\nu\int_\eta^k [\bar{\sigma}_2(\zeta)/(1+\nu\zeta)]d\zeta, \tag{2.10}$$

$$\sigma_{r1} \approx 0, \quad \sigma_{\theta 1} = \sigma_{z1} = \sigma_1 = \int_0^k [\bar{\sigma}_2(\zeta)/(1+\nu\zeta)]d\zeta, \tag{2.11}$$

where $\varepsilon(\zeta) = \varepsilon_\theta(\zeta) = \varepsilon_z(\zeta)$ is the deformation on the inner surface of the substrate.

The boundary effect at growth of a thin-walled tubular substrate with free edges is examined in the article [7]. Using the equation of radial displacements of the long cylindrical shell with uniform moment load on one edge, it is shown that with the exactness of 5% the boundary effect may be considered attenuated at length

$$z_{b.e} = \left[\sqrt{r_1 h_1(1+k)} \;/\; \sqrt[4]{3(1-\mu^2)}\right] \ln\left[20\sqrt{6(1-\mu^2)} \;/\; (1-\mu)\right].$$

Hence, a thin-walled metallic tubular substrate (μ=0.3) may be considered as a long cylindrical shell if the length $l > 6.6\sqrt{r_1 h_1(1+k)}$.

2.3. Hollow spheres with outer coating

The works [84, 9] deal with the determination of residual stresses in coated spheres. In the article [84] using the solution of thermoelastic problem of multilayer hollow sphere the algorithm for the determination of residual stresses in spherical substrate with thick coating is composed for the case of layered coating and substrate. The same problem is solved by integral approach in the paper [9].

Consider the algorithm, presented in the articles [84, 9] for the special case of the homogeneous coating and substrate:

$$\bar{\sigma}_2 = -[E_1/9(1-\mu_1)(1-\mu_2)][(a_o\xi^3-a_1)/\xi^2]d\varepsilon(\xi)/d\xi, \qquad (2.12)$$

$$\sigma^*_{r2} = -2(a_o-a_1/\rho^3)S(\rho), \quad \sigma^*_{\theta 2} = -(2a_o+a_1/\rho^3)S(\rho), \qquad (2.13)$$

$$\sigma_{r1}=-6(1-\mu_2)(1-k_o^3/\rho^3)S(1), \quad \sigma_{\theta 1}=-3(1-\mu_2)(2+k_o^3/\rho^3)S(1). \qquad (2.14)$$

In these formulas

$$a_o = \vartheta[2(1-2\mu_1)+(1+\mu_1)k_o^3]+(1+\mu_2)(1-k_o^3),$$

$$a_1 = \vartheta[2(1-2\mu_1)+(1+\mu_1)k_o^3]-2(1-2\mu_2)(1-k_o^3),$$

$$S(\rho) = \int_\rho^{k_2} [\bar{\sigma}_2(\xi)\xi^2/(a_o\xi^3-a_1)]d\xi.$$

It is possible to determine the residual stresses in the coating on the outer surface of a hollow sphere with the formulas (2.12) and (2.13) by the deformation $\varepsilon(\xi)$, measured on the inner surface of the substrate. If the initial stress $\bar{\sigma}_2(\xi)$ has been determined by the X-ray method or by the deformation of a thin-walled substrate, it is possible to calculate the residual stresses in the coating and the substrate by the formulas (2.13) and (2.14).

In the case of the solid substrate in the formulas (2.13) and (2.14) $k_o = 0$ must be taken. Note in the special case if $E_1 = E_2 = E$, $\mu_1 = \mu_2 = \mu$ and $k_o = 0$ from expressions (2.13) and (2.14) follow the formulas which in the article [41] have been obtained especially for the X-ray method.

In the case of a coated thin-walled spherical substrate $(h_1, h_2) << r_1$ and basic formulas are simplified to the formulas (2.9)-(2.11) of a coated thin-walled tubular substrate.

3. METHODS FOR DETERMINATION OF INITIAL STRESS

As noted before the methods for determination of initial stress are divided into deformation and force methods. The survey of these deformation methods the theory of which the author has developed will be given below. The methods worked out by the author or with his participation are considered as well (Sect.3.6 and 3.10).

3.1. Method of measuring the deformation parameters of a free strip or plate substrate with unilateral coating

In the case of deformation measuring method the initial stress is calculated by the formula (2.1), in the case of curvature measuring method - by the formula (2.2). On assumption that the distribution of initial stress is linear fractional (1.25), the corresponding expressions for approximation of experimental information are the following:

$$\varepsilon = (2/E_1')\bar{\sigma}_2(0)P_1(\zeta), \quad (3.1)$$

$$\varkappa = (6/E_1'h_1)\bar{\sigma}_2(0)P_2(\zeta), \quad (3.2)$$

where

$$P_j(\zeta) = \int_0^{\zeta} [f_j(\zeta)/f(\zeta)][(1+\nu\zeta)/(1+c\nu\zeta)]d\zeta \quad (j=1,\ 2).$$

In general the values of the function $P_j(\zeta)$ are found by numerical method of definite integral calculation. In the special case, if the elastic constants of coating and substrate are equal ($E_1 = E_2 = E$, $\mu_1 = \mu_2 = \mu$), the integral $P_j(\zeta)$ will be calculated analytically. For example, in the case of deformation measuring method the approximative equation is formed:

$$\varepsilon = [2(1-\mu)\bar{\sigma}_2(0)/cE]\ln(1+c\zeta). \quad (3.3)$$

Note that the calculation of initial stress by the curvature of plate substrate is dealt also in the paper [4] where it is shown that in the article [91] the radii of gyration of the bimetallic and the homogeneous bar are erroneously identified. Using the differential approach the formula is obtained which follows from the expression (2.2) if the curvature is expressed by the deflection of free end of a cantilevered substrate. At the same time it is noted that in a special case while $\mu_1 = \mu_2 = \mu$, the formulas obtained in the articles [75, 82] follow from the corrected formula.

In a very thin coating (film) the residual stresses equal to the initial stress is determined on the assumption that the physico-chemical processes causing the initial deformation take place only after the formation of the coating with the finite thickness [45]. Following from the formulas (2.1) and (2.2) the expressions for calculation of residual stresses in very thin coatings are:

$$\bar{\sigma}_2 = \begin{cases} (E_1'/2k)\varepsilon, & (3.3) \\ (E_1'h_1/6k)\varkappa. & (3.4) \end{cases}$$

Comparing with the well-known Stoney's formula [57] the formula (3.4) takes into consideration the biaxial state of stress. The latter is shown as the factor $1/(1-\mu_1)$ in the formula (3.4) [82].

3.2. Method of measuring the deformation parameters of a plate substrate with slipping edges and unilateral coating

In the case of a plate substrate with slipping edges the linear displacements of substrate edges are possible only on the plane of the substrate, the edge moments (1.12) are balanced with reaction moments of redundant constraints and thus the coated substrate deforms momentlessly during the coating process, i.e. only due to the action of edge forces (1.12) [6]. As a thin-walled tubular substrate deforms also in the same conditions in the case of unilateral growth of the plate substrate with slipping edges, the expression (2.9) between initial stress and deformation measured of the free surface of the substrate is valid.

The initial stress can be determined by the curvature, arising after releasing the substrate (removing the redundant constraints). The corresponding formula from the article [6] could be expressed as follows:

$$\bar{\sigma}_2=(E_1'h_1/6)\{[f(\zeta)/f_2(\zeta)]d\varkappa(\zeta)/d\zeta+3\nu[f_2(\zeta)/(1+\nu\zeta)]\varkappa(\zeta)\}. \quad (3.5)$$

This formula differs from that in the monograph [90] as in deduction the edge moment is by mistake calculated proceeding from the distribution of initial stress. If we determine the edge moment before releasing the substrate, considering the distribution of residual stresses following the methods of the work [90] we obtain the formula (3.5).

3.3. Method of measuring the deformation parameters of a plate substrate with fixed edges and unilateral coating

At the coating process of a plate substrate with fixed edges the edge forces and moments (1.12) are balanced by the reaction forces and moments owing to which no additional stresses will arise in coating and substrate. The series of substrate in the identical conditions up to the different thickness h are coated to determine the distribution of initial stress in coating. Then the deformation or curvature on the free surface of substrate generated by release of the edges is measured.

The general treatment of the curvature measuring method is given in the paper [15] where the elastic parameters of the substrate and coating are supposed to change arbitrarily through the thickness. Volterra's first kind integral equation is obtained for the determination of initial stress

$$\int_0^h \bar{\sigma}_2(z)[e(h)+z]dz = D(h)\varkappa(h), \tag{3.6}$$

where D(h) is flexural rigidity of a coated substrate.

The solution of integral equation (3.6) is found by reduction to differential equation. In a special case if elastic parameters of coating and substrate are constant, the solution may be expressed in the form

$$\bar{\sigma}_2=\frac{E_2'h_1}{6}\left[\frac{f(\eta)}{\nu f_2(\eta)}\frac{d\varkappa(\eta)}{d\eta}+\frac{4\omega(\eta)}{f_2(\eta)}\varkappa(\eta)+2\int_0^\eta\frac{f(\zeta)}{f_2^2(\zeta)}\varkappa(\zeta)d\zeta\right], \tag{3.7}$$

where

$$\omega(\eta) = 1+3\eta+3\eta^2+\nu\eta^3.$$

Presuming the linear fractional distribution of the initial stress (1.25) a formula for the approximation of experimental information has been obtained

$$\varkappa = [6\bar{\sigma}_2(0)/E_1'h_1\nu^2c^3f(\eta)]\{c\nu[c\nu-2(1-c)+\nu(3c-2)\eta]\eta+$$
$$+ (1-c)[2-c\nu+ \nu(2+c\nu\eta)\eta]\ln(1+c\nu\eta)\}. \tag{3.8}$$

Note the curvature measuring method of strip substrate with fixed ends is for first time described in the article [56], where the state of stress is supposed to be uniaxial, initial stress - constant and elastic parameters of coating and substrate - equal. In the article [39] the method is generalized for the case of different elastic constants of coating and substrate. The author has dealt with the case of coating of the plate substrate with fixed edges in the articles [82, 6].

The general treatment of the deformation measuring method is presented in the work [8] where the problem of determination of initial stresses is also reduced to the solution of Volterra's first kind integral equation. From the solution obtained by reduction to differential equation there follows the formula for the calculation of initial stress in the case of constant elastic parameters of coating and substrate

$$\bar{\sigma}_2=E_2'\left[\frac{f(\eta)}{2\nu f_1(\eta)}\frac{d\varepsilon(\eta)}{d\eta}+2\frac{\omega(\eta)}{f_1(\eta)}\varepsilon(\eta)+3(1+\eta)\int_0^{\eta}\frac{f(\zeta)}{f_1^2(\zeta)}\varepsilon(\zeta)d\zeta\right]. \qquad (3.9)$$

The formula for the approximation of experimental information is obtained presuming the linear fractional distribution of initial stress

$$\varepsilon=[\bar{\sigma}_2(0)/cE_1'f(\eta)]\{\eta[2\upsilon(\eta)+3\eta\chi(\eta)]+[2(c-1)/c\nu][(1/c\nu)(c\nu\upsilon(\eta)-$$
$$-\ 3\chi(\eta))\ln(1+c\nu\eta)+3\eta\chi(\eta)]\}, \qquad (3.10)$$

where

$$\upsilon(\eta)\ =\ 1-3\nu\eta^2-2\nu\eta^3,\quad \chi(\eta)\ =\ 1+2\nu\eta+\nu\eta^2.$$

3.4. Method of measuring the longitudinal deformation of a straight strip substrate with bilateral coating

According to this method for the determination of initial stress the longitudinal deformation of the straight strip substrate is measured during the simultaneous coating of two sides, while the substrate is prestressed either with gravity force [92] or an elastic element (e.g. semiconductor gage) [34, 35]. The formula for the calculation of initial stress used in the articles [34,35] has been deducted for the calculation of average stress in thin coatings.

In the paper [18] using the equilibrium equations, generalized Hooke's law and continuity conditions of deformations the system of equations is composed where the formula for calculation of initial stress is obtained

$$\bar{\sigma}_2=\frac{1}{2}\left\{\frac{E_1'}{l_1}(1+2\nu\zeta)+\frac{C}{(1-\mu_1)bh_1}\left[\frac{l_0}{l_1}(1+2\nu\zeta)+\frac{\lambda+2\nu\zeta}{\lambda+2\vartheta\zeta}\right]\right\}\frac{d\Lambda(\zeta)}{d\zeta}, \qquad (3.11)$$

where l_0 is the length of region of the substrate without coating, l_1 - length of the coated region, b - width of the substrate, C - rigidity of the elastic element (if the pre-stressing is created by the gravity force, then C = 0), $\Lambda(\zeta)$-axial displacement of the movable end of the substrate.

In the case of a very thin coating the initial stress may be calculated by approximate formula

$$\bar{\sigma}_2 = (E_1'/2)[1/l_1 + C(l_0 + l_1)/E_1 b l_1 h_1] d\Lambda(\zeta)/d\zeta \qquad (3.12)$$

obtained by limiting process h → 0 from the formula (3.11).

As an example of application the distribution of initial stresses calculated by formula (3.12) in cobalt coating of platinum substrate is given in Appendix (Example A.2).

3.5. Method of measuring the longitudinal deformation of a round wire substrate

In the works [83, 54] it is shown that the initial stresses in coatings may be determined by the longitudinal deformation of cylindrical substrate (see Sect.2.2). In the paper [29] more general method for the determination of initial stresses are given. Unlike the articles [83, 54] it takes into consideration the prestressing of the substrate by an elastic element [44, 64].

By using the solution of Lamé's problem for a long cylinder with uniform radial load, the continuity conditions of deformation and equilibrium equation the basic formula of the method is obtained:

$$\bar{\sigma}_2' = -\frac{E_1}{2(1-\mu_2) l_1 \psi_2(h)} \left\{ \psi_1(h) + \frac{C}{\pi E_1}\left[l_1 + \frac{l_2 \psi_1(h)}{r_1^2 - r_0^2} \right] \right\} \frac{d\Lambda(h)}{dh}. \qquad (3.13)$$

In this formula C is rigidity of the elastic element,

$$\psi_1(h) = r_1^2 - r_0^2 + \vartheta\{1 + 2(\mu_1-\mu_2)^2 r_1^2 / [2(1-\mu_2^2) r_1^2 +$$
$$+ (1-\mu_2^2+\gamma)(2r_1+h)h]\}(2r_1+h)h,$$

$$\psi_2(h) = \{1 - 2(\mu_1-\mu_2)(1+\mu_2) r_1^2 / [2(1-\mu_2^2) r_1^2 +$$
$$+ (1-\mu_2^2+\gamma)(2r_1+h)h]\}(r_1+h),$$

where

$$\gamma = \vartheta(1-\mu_1^2)[(r_0^2 + r_1^2)/(r_1^2 - r_0^2) - \mu_1/(1-\mu_1)] + \mu_2(1+\mu_2).$$

Some special cases of the method are thoroughly dealt in the work [29]. Here note, in the case of solid substrate in the formula (3.13) $r_o = 0$ and of prestressing by the gravity $C = 0$.

3.6. Method of measuring the deformation of a thin-walled tubular substrate

The deformation measuring method of a tubular substrate is orientated to the use of strain gages. The idea of the method worked out by the author is given in the article [83]. Determination of initial stresses by the deformation, measured on the inner surface of a thin-walled tubular substrate is dealt in the articles [2, 5, 85, 13] while the most general treatment in the case of elastic parameters of coating and substrate changing arbitrarily through the thickness using integral method has been presented in the report [85].

In the case of constant elastic parameters of coating and substrate, the initial stress is calculated by the formula (2.9). Integrating this equation as a differential equation with respect to the deformation $\varepsilon(\zeta)$, presuming that the initial stress retains the initial value $\bar{\sigma}_2(0)$ and introducing in the result the dimensionless parameter c in order to arise the approximation accuracy, we obtain the expression for the approximation of the experimental information [2]

$$\varepsilon = -[\bar{\sigma}_2(0)/cE_2']\ln(1+c\nu\zeta). \tag{3.14}$$

Taking this expression in differential equation (2.9), we get the linear fractional function (1.25) for describing the initial stress distribution.

3.7. Method of measuring the inner surface displacement of a thin-walled spherical substrate

Measuring the radial displacement $u(\zeta)$ on the inner surface during the coating of thin-walled spherical substrate by principle of liquid thermometer [49], the initial stresses may be calculated by the formula

$$\bar{\sigma}_2 = -(E_1'/r_o)(1+\nu\zeta)du(\zeta)/d\zeta, \tag{3.15}$$

which follows from the formula (2.12) on the assumption that $(h, h_1) \ll r_0$.

3.8. Method of measuring the circumferential deformation of a thin-walled ring substrate

The circumferential deformation measuring method of a thin-walled ring substrate was used in the article [73], while the coated substrate was treated as a bimetallic disc when calculating residual stresses [77]. Taking into account that thin-walled ring substrate is better to be treated as a short cylindrical shell, the corresponding formula for initial stress is presented in the articles [20, 31]

$$\bar{\sigma}_2 = -E_1'\{(1+\nu\zeta)/[1+\Phi(\zeta)]\}d\varepsilon(\zeta)/d\zeta. \qquad (3.16)$$

In the latter formula

$$\Phi(\zeta) = \sqrt{\frac{3(1+\mu)f_2^2(\zeta)}{(1-\mu)\quad f(\zeta)}}\;\frac{\text{ch}\lambda^o(\zeta)\sin\lambda^o(\zeta)-\text{sh}\lambda^o(\zeta)\cos\lambda^o(\zeta)}{\text{sh}\lambda^o(\zeta)\text{ch}\lambda^o(\zeta)+\sin\lambda^o(\zeta)\cos\lambda^o(\zeta)},$$

$$\lambda^o(\zeta) = \lambda(\zeta)/2,\ \lambda(\zeta) = b\sqrt[4]{[3(1-\mu^2)/h_1^2](1+\nu\zeta)^2/R^2(\zeta)f(\zeta)},$$

$$R(\zeta) = R_0 + \nu h_1(1+\zeta)\zeta/2(1+\nu\zeta),$$

where b and R_0 are the width of the ring substrate and the radius of the middle surface, respectively.

The formula (3.16) has been obtained solving the axial-symmetric problem of a short cylindrical shell with surface and edge loads (1.11), (1.12) within the framework of technical theory of shells [71] on the assumption that the elastic moduli of coating and substrate are constant and Poisson's ratios equal ($\mu_1 = \mu_2 = \mu$).

3.9. Method of measuring the angular deflection of a helical warped strip substrate with unilateral coating

The method for measuring the angular deflection of free end of strip substrate with the curvilineared axis and unilateral coating is used in stress measuring, above all for its great sensibility [38, 51, 55, 59, 77, 92].

In the papers [22, 30] there is presented the developed theory of the method, which, unlike the theory based on the bar substrate model, proceeds from the model cylindrical shell with curvilineared edges. The bending problem of two-layer shell, loaded with edge moments (1.12) was solved by adding the states of stresses corresponding to the pure bending and edge effect [67, 68], on the assumption that Poisson's ratios of coating and substrate are equal ($\mu_1 = \mu_2 = \mu$). As a result the following formula for calculation of initial stress by angular deflection $\varphi(\zeta)$ of free end of substrate was obtained:

$$\bar{\sigma}_2 = (E_1 h_1 / 12\pi \mathbf{n})[F(\zeta) f(\zeta) / R(\zeta) f_2(\zeta)] d\varphi(\zeta)/d\zeta . \qquad (3.17)$$

In this formula $\mathbf{n}$ is the number of substrate coils,

$$F(\zeta) = \frac{[F_1(\zeta)F_4(\zeta)\sin^2\alpha + F_2(\zeta)F_3(\zeta)]/(1-\mu^2)}{F_1(\zeta)F_3(\zeta) - \beta(\zeta)F_4(\zeta)\sin^2\alpha + (tg^2\alpha)[F_1^2(\zeta) + \beta(\zeta)F_2(\zeta)]/2},$$

where α is helix angle of a substrate coil,

$F_1(\zeta) = 1 - \beta(\zeta)(\sin^2\alpha + \mu\cos^2\alpha)$, $F_2(\zeta) = 1 - \beta(\zeta)(\sin^2\alpha + \mu\cos^2\alpha)^2$,
$F_3(\zeta) = 1 - \beta(\zeta)(1-\mu)\sin^2\alpha$, $F_4(\zeta) = \beta(\zeta)(1-\mu)(\sin^2\alpha + \mu\cos^2\alpha)$,
$\beta(\zeta) = [2/\lambda^*(\zeta)][ch\lambda^*(\zeta) - \cos\lambda^*(\zeta)]/[sh\lambda^*(\zeta) + \sin\lambda^*(\zeta)]$,
$\lambda^*(\zeta) = \lambda(\zeta)\cos\alpha$.

The function $F(\zeta)$ in the formula (3.17) may be considered as a correction factor for the formula obtained on the ground of the bar substrate model. It is shown that for substrate used in practice (λ = 10-13, α = 12°-24°) the range of $F(\zeta)$ is $1.12 \geq F(\zeta) \geq 1.04$. The developed formula (3.17) gives 4-12% greater initial stresses compared with the formulas used earlier [38, 39, 51, 55, 63, 77, 92].

3.10. Method of measuring the deflection of an unclosed ring strip substrate with slipping edges and unilateral coating

The method for the determination of initial stresses in coatings grown by brush-plating is suitable on the technological considerations [16]. For coating the outer surface the substrate is fixed into the mandrel which makes free slipping of the edges possible and thus the momentless deformation of coated substrate as well. The coated substrate is released from the mandrel and the slit increment of the substrate $\delta(\zeta)$ is measured as bending deflection in order to determine initial stresses. Since releasing the coated substrate from the mandrel is equivalent to the loading of the substrate edges by uniform moment (1.20) the determination of initial stresses reduces to solving the bending problem of a short two-layer unclosed cylindrical shell. Solving this problem within the limits of technical theory of shells [68] has enabled to obtain the Volterra's first kind integral equation, from which

$$\bar{\sigma}_2=(E_1h_1/12\pi R_o^2)F_o\{[f(\zeta)/f_2(\zeta)]d\delta(\zeta)/d\zeta+[3\nu f_2(\zeta)/(1+\nu\zeta)]\delta(\zeta)\}. \quad (3.18)$$

Here

$$F_o= (1-\mu^2\beta)/(1-\mu^2)(1-\mu\beta),$$

where

$$\beta = (2/\lambda)(\mathrm{ch}\lambda-\cos\lambda)/(\mathrm{sh}\lambda+\sin\lambda), \quad \lambda = b\sqrt[4]{3(1-\mu^2)/h_1^2R_o^2}.$$

4. THERMAL STRESSES IN COATED PARTS

As usual the coefficients of thermal expansion of coating and substrate are different and the coating process temperature differs from the normal temperature (+20^{o}C), thermal stresses generate in coated parts. The latter combined with the technological residual stresses determine the inherent stress state in coated parts.

In the experimental determination of residual stresses it may appear that the coating temperature differs from that of the deformation parameters were measured. In such cases one has to correct the measurement results considering thermal strains. In addition the solutions of thermoelastic problems may be used for obtaining the relations between deformation parameters of the substrate and initial stress (see Sect.1.2).

Considering above-mentioned, the author has dedicated some papers to solving the thermoelastic problems. In this chapter a short review of these works is given.

4.1. Free plates with unilateral coating

The articles [81, 12, 14] deal with the determination of thermal stresses in nonhomogeneous free rectangular bars and plates with arbitrary contour. The paper [14] includes the most general treatment where the thermoelastic problem of bars and plates with arbitrarily changing elastic parameters and temperature through thickness by semiinverse method is solved. According to the solution a plate bends spherically, a beam - circularly. The solution includes, as a special case, a well-known result for a homogeneous plate and a beam [37].

In the case of constant elastic parameters and uniform temperature change ΔT, the curvature of coated substrate is given by the expression

$$\varkappa = 6\nu k(1+k)(\alpha_1-\alpha_2)\Delta T/f(k)h_1, \tag{4.1}$$

where

$$f(k) = 4\nu k(1+k)^2+ (1-\nu k^2)^2. \tag{4.2}$$

Thermal stresses in the coating and the substrate:

$$\sigma_2 = E_2'\{1+ \nu k[3k+ 4k^2- 6(1+k)\eta]\}(\alpha_1- \alpha_2)\Delta T/f(k), \tag{4.3}$$

$$\sigma_1 = -E_2'k[4+3k+ \nu k^3+ 6(1+k)\eta](\alpha_1- \alpha_2)\Delta T/f(k). \tag{4.4}$$

The maximum stresses in the coating and the substrate arise at interface ($\eta=0$) [12].

The formulas (4.1) and (4.2) show that in the case of fixed summary thickness $h_1+ h_2$, the thermosensibility of bimetallic plate is maximal if $\nu k^2= 1$. This condition includes the Villarceau's condition as a special case.

Note Timoshenko's well-known formula [60] results from the expression (4.1) in the special case $\mu_1= \mu_2= \mu$. Formulas (4.1), (4.3) and (4.4) for normal thermobimetal ($\nu k^2= 1$) simplify [23]:

$$\varkappa = 3(\alpha_1-\alpha_2)\Delta T/2(1+k)h_1, \tag{4.5}$$

$$\sigma_2 = E_2'(2k- 3\eta)(\alpha_1- \alpha_2)\Delta T/2(1+k), \tag{4.6}$$

$$\sigma_1 = -E_1'(2+3\eta)(\alpha_1- \alpha_2)\Delta T/2(1+k). \tag{4.7}$$

Researches made by the holographic method show (see Sect. 5.2) that thermobimetallic strips with the relative thickness $(h_1+h_2)/b \le 1/20$ and width $b/l \ge 1/5$ bend spherically by uniform temperature change. As the relative thickness and width of the thermobimetallic springs used in the instrument engineering correspond to the ones of analyzed strips, it is advisable at calculating to take them not as strips as usually done [66], but as plates. At the same time, note the curvature formula of normal thermobimetal (4.5) does not include elastic constants and so it gives the same results for a strip and a plate. As regards to stress, according to the formulas (4.6) and (4.7) the stresses in the layers of a plate are $1/(1-\mu_2)$ and $1/(1-\mu_1)$ times greater than in the corresponding layers of a strip.

4.2. Coated circular cylinders

In the article [83] the axial symmetric thermoelastic problem of a multilayer (piecewise homogeneous) circular cylinder has been solved. According to the obtained algorithm from the system of linear equations the radial stresses on the interfaces of layers and the longitudinal deformation will be found and then the stresses in the layers will be calculated. Uniform temperature change ΔT causes the following stresses in the coated hollow cylinder [12]:

$$(\sigma_{r2},\sigma_{\theta 2}) = [p/(k_2^2-1)](1 \mp k_2^2/\rho^2),$$
$$\sigma_{z2} = E_2(\varepsilon_z-\alpha_2\Delta T)-2\mu_2 p/(k_2^2-1), \tag{4.8}$$

$$(\sigma_{r1},\sigma_{\theta 1}) = [p/(1-k_o^2)](1 \mp k_o^2/\rho^2),$$
$$\sigma_{z1} = E_1(\varepsilon_z-\alpha_1\Delta T)+2\mu_1 p/(1-k_o^2). \tag{4.9}$$

In these formulas

$$p = (c_{14}c_{20}-c_{10}c_{22})/(c_{10}c_{21}-c_{12}c_{20}),$$
$$\varepsilon_z = (c_{12}c_{22}-c_{14}c_{21})/(c_{10}c_{21}-c_{12}c_{20}),$$

where

$$c_{14} = [(1+\mu_2)\alpha_2-(1+\mu_1)\alpha_1]\Delta T/2,$$
$$c_{20} = [E_1(1-k_o^2)+E_2(k_2^2-1)]/2, \qquad c_{10} = (\mu_1-\mu_2)/2,$$
$$c_{22} = -[E_1(1-k_o^2)\alpha_1+E_2(k_2^2-1)\alpha_2]\Delta T/2, \qquad c_{21} = \mu_1-\mu_2,$$
$$c_{12} = \frac{1+\mu_1}{E_1}\left[\frac{1}{2}-\frac{1-\mu_1}{1-k_o^2}\right]-\frac{1+\mu_2}{E_2}\left[\frac{1}{2}+\frac{1-\mu_2}{k_2^2-1}\right].$$

Given formulas include formulas of coated solid cylinder as a special case ($k_o = 0$).

In the case of a coated thin-walled tubular substrate radial stresses are negligible, circumferential and longitudinal stresses are expressed as follows [12]:

$$\sigma_{\theta 2} \approx \sigma_{z2} = -E_2'(\alpha_2-\alpha_1)\Delta T/(1+\nu k),$$
$$\sigma_{\theta 1} \approx \sigma_{z1} = E_2'(\alpha_2-\alpha_1)\Delta Tk/(1+\nu k). \qquad (4.10)$$

4.3. Coated spheres

The thermoelastic problem of a central symmetric multilayer (piecewise homogeneous) hollow sphere is solved in the paper [84]. According to the gained algorithm the radial stresses on the interface of layers are found from the system of equations with three-diagonal matrix and then the stresses in layers are calculated.

Uniform temperature change causes the following stresses in a coated hollow sphere [12]:

$$\sigma_{r2}= [p/(k_2^3- 1)](k_2^3-\rho^3)/\rho^3,$$
$$\sigma_{\theta 2}= -[p/2(k_2^3- 1)](k_2^3+2\rho^3)/\rho^3, \qquad (4.11)$$

$$\sigma_{r1}= [p/(1-k_o^3)](\rho^3-k_o^3)/\rho^3,$$
$$\sigma_{\theta 1}= [p/2(1-k_o^3)](k_o^3+2\rho^3)/\rho^3. \qquad (4.12)$$

In these formulas

$$p = \frac{2E_1(\alpha_2-\alpha_1)\Delta T}{[2(1-2\mu_1)+(1+\mu_1)k_o^3]/(1-k_o^3)+[2(1-2\mu_2)+(1+\mu_2)k_2^3]/(k_2^3-1)\vartheta}.$$

For coated solid sphere $k_o= 0$ is taken. Radial stresses of a coated thin-walled spherical substrate are negligible, but circumferential stresses are equal to the corresponding ones of a thin-walled tubular substrate (4.10).

5. SOME RESULTS OF THE EXPERIMENTS

In this chapter a summary of the author's experimental researches on modelling continuous growth of coating in layers, verifying the deformation of a strip substrate and determination of initial stresses in galvanic steel coatings is presented.

5.1. Modelling of residual stresses by the photoelastic method

As noticed above (see Sect. 1.1) mechanical methods for the determination of residual stresses are based on the model of continuous growth in layers. To verify this model an unilateral coating growth of a rectangular bar in the case of constant initial stress was modelled by the photoelastic method [25, 33].

The width of the model, made of epoxide resin ЭД-5 was 9.5 mm, the height of the substrate 25.4 mm and the length 220 mm. The modelling part of the coating with the height h_2= 25.3 mm was obtained by consecutively glueing eleven strips of 2.2x9.5 mm prestressed with initial stress $\bar{\sigma}_2$= 2.7 MPa. For technological reasons momentless state was realized in the middle region of the model during its making. For that purpose the model in the test apparatus was loaded according to the scheme of the beam with two cantilevers so that during the growth the region of the model between supports was kept straight by reducing the deflection to zero.

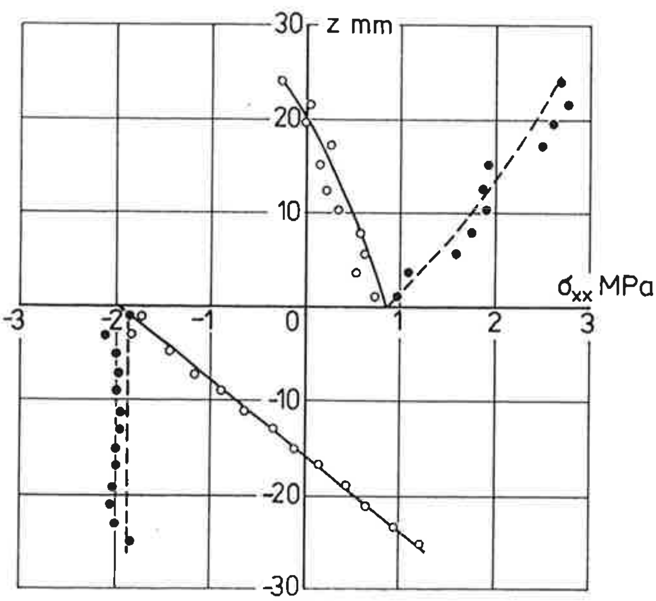

Fig.5.1. Distribution of normal residual stresses σ_{xx} in the middle section of the model. Loaded model:--- theory, • photoelastic analysis. Free model: —— theory, ∘ photoelastic analysis

The distribution of normal residual stresses in the middle section of the model (Fig.5.1) was determined before and after releasing the ends from the load. The calculated values were obtained by the formulas based on the model of continuous coating growth. The distribution of shear residual stresses (Fig. 5.2) was determined on the end region of the free model, close to the interface between the substrate and the coating after cutting off the cantilevers. The theoretical values were obtained using the solution of a thermoelastic problem on a two-layer bar [40] as well by the finite elements method.

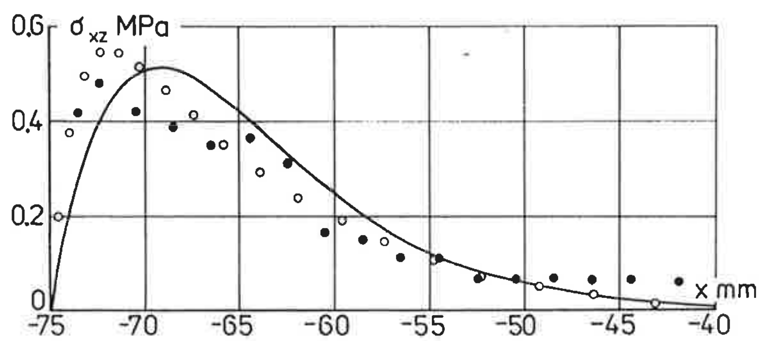

Fig.5.2. Distribution of shear residual stresses σ_{xz} in the longitudinal section z = −1.1 mm of the end region of the free model: ——— theory, • finite element analysis, ∘ photoelastic analysis

From Figs.5.1 and 5.2 it follows that the theoretical and experimental data satisfactorily coincide. The area of distribution of the edge effect is approximately equal to the height of the model. Thus, the results of the modelling confirm the validity of the model of continuous growth in layers and the well-known Saint-Venant's principle.

5.2. Deformation analysis of straight strip substrates

Using the method of measuring the deformation parameters unilaterally coated plate substrate, a rectangular plate is ordinarily used as a substrate. If the width of the substrate is big enough compared with the length, the substrate is observed as a plate which deforms spherically during the coating process [19]. A straight strip substrate (a narrow rectangular plate), the width of which is small compared with the length, but thickness is smaller than width is very often used as a substrate and the substrate cannot be taken as a bar. In the case of such strip substrate the question arises about the character of its deformation, i.e. the problem is if the substrate can be taken as a plate or is it necessary to form a new theory which takes into consideration the relative width and thickness.

In the article [19] within the scope of technical theory of plates it is shown that the deformation of the plate substrate in growing or removal process is similar to the deformation of a bimetallic plate which is caused by uniform temperature change. Since the experimental study of a bimetal specimen deformation caused by temperature change is technically easier, thermobimetal analogy was used to explain the effect of the width of the plate substrate on its deformation during the growing or removal coating.

In the article [19] there are given the results of determination of curvature in longitudinal and lateral direction of the strips made of normal thermobimetal ТБ-1523 (thickness $h = 0.6$ mm, length $l = 50$ mm) [66]. For the determination of curvature a specimen was fixed immovably on the short midline of one side. The angle of rotation in the middle of the opposite side, caused by temperature change, was measured by an optical goniometer. Dependence of the curvatures $\varkappa$, calculated by the angle of rotation on the relative width b/l of the specimen is presented in Fig.5.3 which shows the data lie in the band of admissible deviations of the theoretical values. As a result the bending of the strip can be considered spherical, if its relative width $b/l \geq 0.05$.

The effect of the fixing length of the strip was also observed. It appeared the fixation length does not have any essential effect on bending of the strip with the relative width $b/l \leq 0.3$.

As the determination of curvature of bimetal strip the angle of rotation of the edge assures the spherical bending of the strip only indirectly, the deformation of the strip was examined by the method of interference holography [21, 23] as well. The analysis of the interferograms of the deformations, caused by the uniform cooling of the strips, made of the bimetal TB-1523 (thickness h = 0.6 mm, length l = 50 mm), has shown that the strips of interference were circular, i.e. the surface of the specimen is deformed into the surface of revolution. For the refinement of the surface the displacements measured on the longer axis of symmetry were approximated with a quadratic polynomial which in case of small deflections corresponds quite precisely to the assumption that the specimen bends spherically.

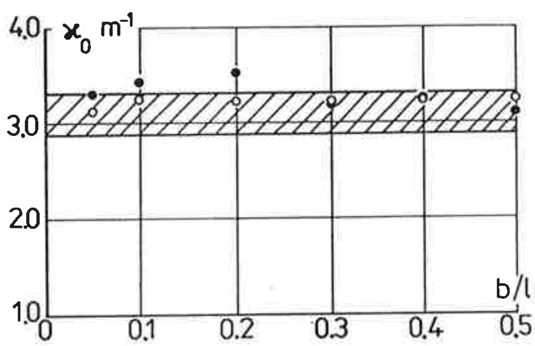

Fig.5.3. Curvature of a bimetal strip depending on the relative width: band of admissible deviations of theoretical values (hatched), experimental values of longitudinal curvatures (∘) and lateral curvatures (•)

The polynomial constants - displacement w_o and angle of rotation φ_o at the origin of the coordinates (in the middle of the specimen) and the curvature $\varkappa$ in the middle region with length l_o were determined by the least-squares method. The results obtained by three specimens are given in Table 5.1, where the quadratic mean deviation of approximative function and experimental results δ are shown. As the latter is of the same order with measuring accuracy (~79 nm), the approximation with quadratic polynomial may be considered optimal and surface of the specimen spherical.

Table 5.1. Deformation parameters of bimetal specimens

b (mm)	b/l	ΔT (°C)	l_o (mm)	$\varkappa \cdot 10^5$ (mm^{-1})	w_o (μm)	$\varphi_o \cdot 10^4$ (rad)	δ (nm)
7.8	0.16	-0.9	43.5	$\frac{4.16}{4.05}$	9.76	0.233	142
17.8	0.36	-1.0	36.6	$\frac{4.65}{4.50}$	7.72	0.081	37
25.0	0.50	-0.9	47.5	$\frac{4.08}{4.05}$	11.48	0.089	42

Note. Experimentally obtained curvature values are shown in the numerator, the values calculated by the formula (4.5) - in the denominator

Thus, proceeding from the results gained by the holographic method we can assure that coated strip substrate with the relative thickness $h/b \leq 0.08$ and the relative width $b/l \geq 0.16$ bend spherically during the growth or removal of the coating. Deformation study of the straight strip substrate during its unilateral covering with the colloid thin film [86] has confirmed this result.

5.3. Deformation analysis of curvilinear strip substrates

It is known that uniform temperature change of the shell, nonhomogeneous in the direction of thickness, causes the same bending deformation as the uniform edge moment [65]. This analogy has allowed to model the bending of curvilinear strip substrate during the coating by bending of the curvilinear thermobimetal strip at uniform heating and thus experimentally check up the bending theory of coated curvilinear strip substrate advanced in the article [22], according to which the edge moment equivalent to uniform temperature change ΔT causes the angle of rotation

$$\varphi_T = 3\pi n R_o(\alpha_1-\alpha_2)\Delta T/F(k)(1-\mu)(h_1+h_2) \qquad (5.1)$$

of the free end of curvilinear strip, made of normal thermobimetal. In the formula (5.1) F(k) is the parameter defined in Sect.3.9.

Table 5.2. Angle of rotation of free end of curvilinear thermobimetal strips at uniform heating

Specimen no.	Geometric parameters		Angle of rotation φ_T (rad)		Discrepancy (%)
	Width b (mm)	Angle of slope α (deg)	Theory	Experiment	
1	7.1	10.0	0.208	0.211	+1.4
2	8.9	14.7	0.217	0.222	+2.3
3	10.1	14.0	0.226	0.228	+0.9
4	10.1	18.9	0.228	0.230	+0.9
5	10.1	22.4	0.231	0.235	+1.7
6	11.0	13.5	0.227	0.228	+0.4
7	13.4	19.3	0.245	0.242	-1.2
8	15.5	19.8	0.255	0.248	-2.7

The specimens were made of thermobimetal ТБ-1523 with the thickness of 0.78 mm. The number of coils of all specimens n=4 and radius of middle surface R_o=7,5 mm. The angle of rotation of the specimens free end while heating in the thermostat was measured by an optical goniometer. The comparison of theoretical and experimental values of the angle of rotation for $\Delta T = 70^\circ C$ and $\alpha_1 - \alpha_2 = 18 \cdot 10^{-6} 1/{}^\circ C$ is presented in Table 5.2.

As shown in the table the discrepancy of theoretical and experimental data does not exceed 2.7%. Such result allows to evaluate the bending theory of the coated curvilinear strip substrate [22] as quite perfect and at the same time recommend the formula (5.1) for calculating curvilineared strip springs of bimetal thermometers which at present are calculated on the ground of the model of a curvilinear bar [66].

5.4. Study of residual stresses in galvanic steel coatings

Residual stresses in galvanic steel coatings used for restoration of machine parts have been studied by the deformation measuring method of thin-walled tubular substrate [2, 5,85]. The cathodes with the inner radius r_o=15 mm, the thickness h_1=1mm, the length of the coated region 130mm were used. The deformation was measured by a wire strain gage glued on the inner surface. The coating was carried out in the bath where the rotating anodes and automatically operating heating system were used for keeping uniform thickness and temperature of the electrolyte.

The coatings with the thickness of 0.47 - 0.52 mm were deposited from the electrolyte (g/l): iron chloride 500, sodium chloride 100, manganese chloride 5, free hydrochloride acid 0.8-1.0. The coating temperatures were 90, 92.5 and 95°C, current densities - 1.5 and 2 kA/m^2.

For the determination of elastic constants of coating thin-walled tubular specimens with the inner diameter of 9 mm, the thickness 0.5-0.7 mm, the length of the coated region of 90 mm were used. The specimens were obtained by the deposition of thin etching-safe coating and of the observed coating on a thin-walled tubular substrate and by following etching of the substrate metal from the coated region. The modulus of elasticity was determined by the tensile test. The Poisson's ratio was calculated by modulus of elasticity and shear modulus from torsion test. In the limits of experimental exactness all coating conditions used E_2=200 GPa, μ_2= 0.3 was obtained.

In the first experiments the circumferential and longitudinal deformations were measured in order to explain the character of the state of stress in coating. The analysis of the eight experiments has shown that these deformations are practically equal, i.e. the assumption according to which the biaxial state of stress with equal principal stresses arises in the superficial layer of coating (see Sect. 1.1 and 2.2) is valid. Since the glueing of the strain gages measuring the longitudinal deformation is easier, in remaining experiments the average longitudinal deformation of every cathode was measured by four gages.

The measured deformations were approximated by the logarithmic expression (3.14). The parameters c and $\bar{\sigma}_2(0)$ were determined using the computer program which was based on the method of averages [76].

The analysis has shown that the variation extent 0.41

of the parameter c, obtained under the same conditions in the two experiments, is practically equal to the variation extent of the parameter c respective to different coating conditions. Considering this, the parameter c was taken as equal for all researched coating regimes with the arithmetical mean c = 1.8 gained from all the experiments.

The values of the parameter $\bar{\sigma}_2(0)$ are presented in Table 5.3. As we can see the initial value $\bar{\sigma}_2(0)$ of initial stress, which in the case of the parameter c>1 is the greatest value of the initial stress in the coating, does not depend much on substrate metal. But this parameter is essentially influenced by coating temperature and current density. As the temperature rises $\bar{\sigma}_2(0)$ decreases, but at current density being increased it also increases.

Table 5.3. Initial values of the initial stress of galvanic steel coatings

Cathode material	Coating regime		Initial value of initial stress $\bar{\sigma}_2(0)$ (MPa)
	Temperature (°C)	Current density (kA/m^2)	
Steel 15	90	1.5	268
	90	2.0	300
	92.5	2.0	301
	95	2.0	92
Steel 45	90	1.5	282
	90	2.0	345
	92.5	2.0	265
	95	2.0	103
Copper	90	1.5	286
	90	2.0	330
	92.5	2.0	260
	95	2.0	109

As an example in Appendix (Example A.3) the distributions of initial and residual stresses in a galvanic steel coating of the thin-walled tubular cathode are presented.

CONCLUSIONS

1.General algorithm for determination of residual stresses in coated parts is composed. The algorithm is universal and allows to determine residual stresses on coating growth or removal by deformation parameters, measured on the free surface of the substrate or on the moving surface of the coating. The algorithm may be used when the initial stress is determined by surface physics method or by measuring deformation parameters of a thin-walled substrate.

2. The algorithms for determination of residual stresses in multi-layer rectangular bars, plates, cylinders and spheres with thick multi-layer coating is composed.

3. The theory of methods for determination of initial stresses is developed as follows:

a) in the deformation-parameter measuring method of unilaterally coated strip or plate substrate, coating and substrate are considered non-homogeneous and the state of stress biaxial;

b) in longitudinal deformation measuring method of straight strip substrate with bilateral coating, biaxial character of state of stress and possibility to prestress the substrate with a elastic element are considered;

c) for the inner surface displacements measuring method of the thin-walled spherical substrate (Mills' method) the formula for calculation of initial stress by radial displacement of substrate inner surface is deduced;

d) substrate is treated as a short cylindrical shell in the theory of circumferential deformation measuring method of thin-walled ring substrate;

e) substrate is treated as a cylindrical shell with curvilineared edges in deduction of formula for the method of measuring the angular deflection of helical warped strip substrate.

4. For determination of initial stresses in galvanic coatings the deformation measuring method of thin-walled tubular substrate is elaborated and for determination of initial stress in tampon-galvanic coatings the deformation parameter measuring method of a thin-walled cut ring sub-

strate is elaborated.

5. The methods for determination of thermostresses and deformations in strip and plate substrate with unilateral coating are perfected, considering that coating and substrate are non-homogeneous and the state of stresses has biaxial character. The algorithms for determination of thermostresses and deformations in multi-layer cylinders and spheres with multi-layer coating are composed.

6. Residual stress distribution in unilateral coating of the straight bar substrate, determined on the ground of the model of continuous layer growth of coating, is verified by the photoelastic method.

7. Distribution of shear residual stresses in the end region of unilaterally coated straight strip substrate is determined theoretically by thermobimetal analogy, numerically using the finite-element method and experimentally by photoelastic method.

8. Using thermobimetal analogy it has been shown experimentally that at unilateral coating growth or removal a strip or plate substrate bends spherically.

9. Pure bending theory of helical warped strip substrate with unilateral coating is experimentally verified using thermobimetal analogy.

10. Residual stresses in thick galvanic steel coatings are investigated by the deformation measuring method of a thin-walled tubular substrate. It is shown that for description of initial stress distribution in galvanic coating, two-parametric linear fractional function is suitable.

To improve the results obtained from the thesis, the development of theory and experimental techniques for the large deformations of thin-walled substrate may be considered.

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APPENDIX: EXAMPLES OF APPLICATION

Example A.1. Residual stresses in a boron fibre

In the Fig.A.1 there is shown the distribution of residual stresses in boron fibre, which core radius $r_1 = 8.1\ \mu m$, outer radius $r_2 = 51\ \mu m$, $E_1 = 669$ GPa, $E_2 = 393$ GPa, $\mu_1 = \mu_2 = 0.21$. The stresses are calculated by the longitudinal deformation $\varepsilon_z(\xi)$ taken from the article [36] and previously smoothed by cubic splines. As seen we obtain abased values for longitudinal stresses on assumption of linear stress state, at which the greatest difference is at the interface of core and boron layer.

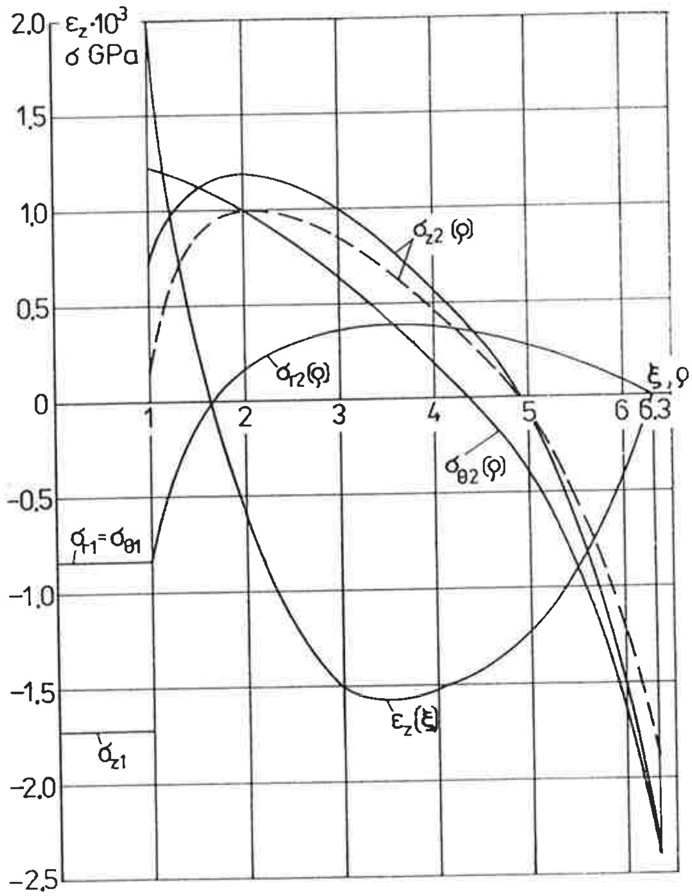

Fig.A.1. Longitudinal deformation ε_z measured during the etching of boron fibre and distribution of residual stresses σ_{r2}, $\sigma_{\theta 2}$, σ_{z2} determined by this deformation. Dash-line shows the distribution of stresses obtained on assumption of uniaxial stress state

Example A.2. Initial stresses in a cobalt coating

In Fig.A.2 the distribution of initial stresses in bilateral cobalt coating (h_2 = 210 nm) of the platinum substrate (h_1= 25 μm, b =12.7 mm, l_1=44 mm, l_o=0, E_1=167 GPa, μ_1=0.39, C = 3.68 MN/m) is shown.

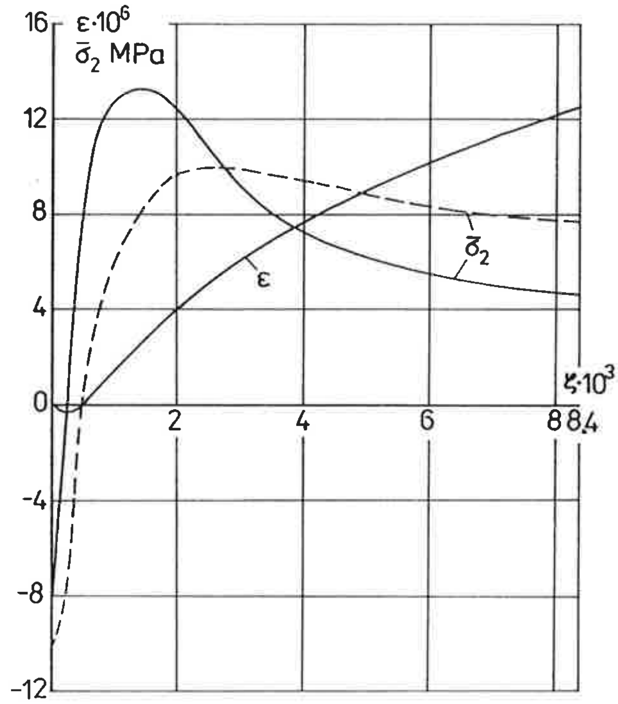

Fig.A.2. Deformation of elastic element measured during the bilateral galvanic deposition of cobalt on the platinum strip substrate and determined according to this initial stress distribution. Dash-line shows the distribution of average initial stress [35]

The initial stress distribution is obtained by the formula (3.12), by the deformation of the gage $\varepsilon=\Lambda/l$ (l is the length of the gage), registered during the coating process of the substrate, prestressed with semiconductor strain gage [35], while the data have been previously smoothed with cubic splines. As seen the distribution of average initial stresses differs essentially from the distribution obtained by the formula (3.12).

Example A.3. Initial and residual stresses in a galvanic steel coating

In Fig.A.3 it is shown the initial and residual stress distributions in galvanic steel coating (h_2=0.49 mm, E_2=200GPa, μ_2= 0.3, α_2= $12\cdot10^{-6}$1/ °C) of thin-walled copper tubular cathode (r_o= 15.01 mm, h_1= 0.98 mm, E_1= 110 GPa, μ_1 = 0.3, α_1=$17\cdot10^{-6}$1/ °C).

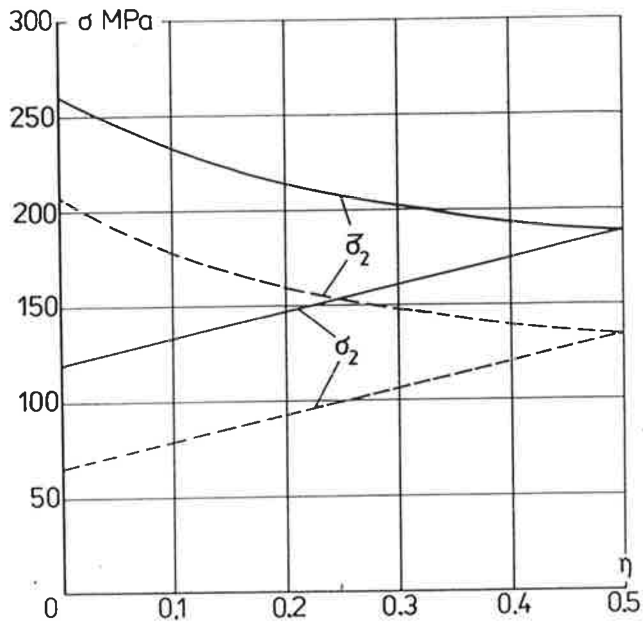

Fig.A.3. Distribution of initial $\bar{\sigma}_2$ and residual σ_2 stresses in galvanic steel coating of thin-walled copper tubular cathode at coating temperature 92.5°C (———) and at normal temperature 20°C (----)

The initial stresses are calculated by the formula (1.25) as follows:

$$\bar{\sigma}_2 = \bar{\sigma}_2(0)(1+\nu\eta)/(1+c\nu\eta). \qquad (A.1)$$

Additional stresses are found by the formula

$$\sigma_2^* = -[\bar{\sigma}_2(0)/c]\ln[(1+c\nu k)/(1+c\nu\eta)],$$

which follows from the formula (2.10) in case of linear fractional initial stress distribution (A.1). For reduction of stress distribution to normal temperature the formula (4.10) was used.