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CONSIDERATION OF THERMAL STRESSES IN EXPERIMENTAL RESIDUAL STRESS ANALYSIS FOR COATINGS

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1. Introduction

Usually, the coefficients of thermal expansion of the coating and substrate are different and the temperature of coating process differs from room temperature (+20°C). Therefore thermal stresses are generated in coated parts.

In experimental analysis of residual stresses it may turn out that coating temperature differs from the temperature at which the deformation parameters of the substrate are measured. In such cases the measurement results have to be corrected taking into account the thermal stresses.

In this study, the equations for correction of the residual stresses in coatings, determined from the measured longitudinal deformations of the wire, and from the tubular and strip substrates [1-3], are presented. Three examples of application are given.

The following symbols are used in the article: E_i, μ_i, α_i - modulus of elasticity, Poisson's ratio, coefficient of thermal expansion of the substrate (i =1) and coating (i = 2).

2. Thermal stresses in a coated long circular cylinder

Consider a long hollow cylinder (substrate) in cylindrical coordinates r, θ, z (Fig. 1). The inner radius of the cylinder is r_0 and the outer radius is r_1. The coating with the outer

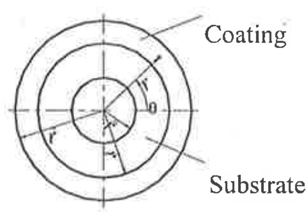

Fig. 1. Hollow cylinder (substrate) with a coating

radius r_2 is deposited on the outer surface of the substrate. The constant temperature change ΔT causes thermal stresses as follows [4,5]:

in the substrate ($r_0 \le r \le r_1$)

$$\left.\begin{matrix}\sigma_{r1}\\ \sigma_{\theta 1}\end{matrix}\right\} = \frac{p}{1-k_0^2}\left(1 \mp \frac{r_0^2}{r^2}\right), \tag{1}$$

$$\sigma_{z1} = E_1(\varepsilon_z - \alpha_1 \Delta T) + \frac{2\mu_1 p}{1-k_0^2}; \tag{2}$$

in the coating ($r_1 \le r \le r_2$)

$$\left.\begin{matrix}\sigma_{r2}\\ \sigma_{\theta 2}\end{matrix}\right\} = -\frac{p}{k_2^2-1}\left(1 \mp \frac{r_2^2}{r^2}\right), \tag{3}$$

$$\sigma_{z2} = E_2(\varepsilon_z - \alpha_2 \Delta T) - \frac{2\mu_2 p}{k_2^2-1}, \tag{4}$$

where

$$p = \frac{c_{14}c_{20} - c_{10}c_{22}}{c_{10}c_{21} - c_{12}c_{20}},\ \varepsilon_z = \frac{c_{12}c_{22} - c_{14}c_{21}}{c_{10}c_{21} - c_{12}c_{20}},$$

$$c_{14} = \frac{1}{2}\left[(1+\mu_2)\alpha_2 - (1+\mu_1)\alpha_1\right]\Delta T,\ c_{20} = \frac{1}{2}\left[E_1(1-k_0^2) + E_2(k_2^2-1)\right],$$

$$c_{22} = -\frac{1}{2}\left[E_1(1-k_0^2)\alpha_1 + E_2(k_2^2-1)\alpha_2\right]\Delta T,\ c_{12} = \frac{1+\mu_1}{E_1}\left(\frac{1}{2} - \frac{1-\mu_1}{1-k_0^2}\right) - \frac{1+\mu_2}{E_2}\left(\frac{1}{2} + \frac{1-\mu_2}{k_2^2-1}\right),$$

$$c_{10} = \frac{1}{2}(\mu_1 - \mu_2),\ c_{21} = \mu_1 - \mu_2,\ k_0 = \frac{r_0}{r_1},\ k_2 = \frac{r_2}{r_1}.$$

As the first example, thermal stresses are computed for a galvanic nickel coating (r_2= 0.49 mm, E_2=178 GPa, μ_2= 0.31, α_2= 7.5×10^{-6} 1/°C) deposited on a copper wire substrate (r_0=0, r_1=0.29 mm, E_1=103 GPa, μ_1=0.34, α_1=18×10^{-6} 1/°C) [1,6].

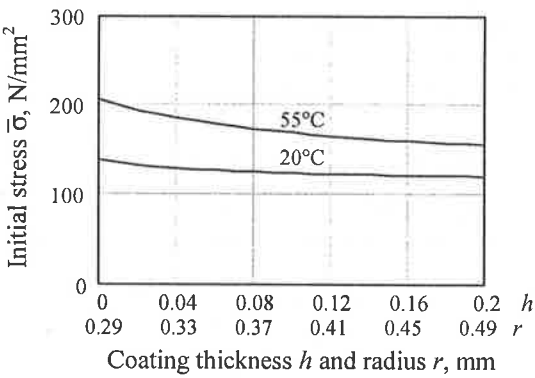

Fig. 2. Distribution of initial stresses in a galvanic nickel coating on a copper wire substrate at a galvanizing temperature of 55°C and at a room temperature of 20°C

Fig. 2 shows the distribution of initial stresses at a galvanizing temperature of 55°C and at a room temperature of 20°C. Initial stresses at 55°C is calculated applying the expression [1,6]

$$\bar{\sigma} = 205\frac{0.20+h}{0.20+1.64h}.$$

Initial stresses at room temperature are obtained by adding the thermal stresses $\sigma_{\theta 2}$ according to formula (3).

As the second example, initial stresses are corrected for a galvanic nickel coating (r_2=1.53mm, E_2=178 GPa, μ_2= 0.31, α_2= 7.5×10^{-6} 1/°C) deposited on a copper tubular substrate (r_0= 0.65 r_1=1.38mm, E_1= 98 GPa, μ_1=0.34, α_1=18×10^{-6} 1/°C) [2]. Fig. 3 shows the distribution of initial stresses at a galvanizing temperature of 55 °C and at a room temperature of 20°C. Initial stresses at galvanizing temperature are calculated from expression [2]

$$\bar{\sigma} = 180\frac{0.15+h}{0.15+1.91h}.$$

Initial stresses at room temperature are determined by adding the thermal stresses $\sigma_{\theta 2}$ according to expression (3).

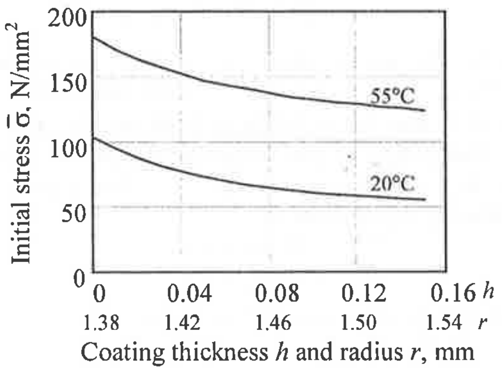

Fig. 3. Distribution of initial stresses in a galvanic nickel coating on a copper tubular substrate at a galvanizing temperature of 55°C and at a room temperature of 20°C

3. Thermal stresses in a bilaterally coated long strip substrate

Consider a long rectilinear strip substrate in rectangular coordinates x, y, z. A cross-section of the coated strip is presented in Fig. 4.

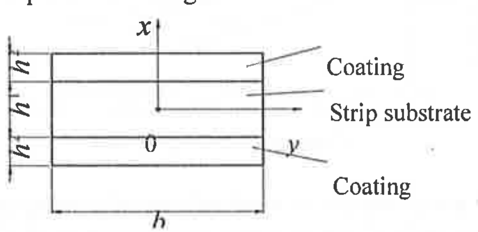

Fig. 4. Cross-section of a bilaterally coated strip substrate

Substrate thickness is denoted by h_1, coating thickness by h_2 and substrate width by b.

Assuming that the stresses in the plane stress state are equal, the basic stress-strain relations of thermoelasticity [7] for the substrate (i=1) and for the coating (i=2) are

$$\varepsilon_i = \frac{1-\mu_i}{E_i}\sigma_i + \alpha_i \Delta T . \tag{5}$$

According to Kirchoff's deformation hypothesis [7], deformations do not depend on the coordinate x, i.e. $\varepsilon_1=\varepsilon_2$=const. An equilibrium equation can be written as

$$\sigma_1 h_1 + 2\sigma_2 h_2 = 0 . \tag{6}$$

From equations (5) and (6), we will find thermal stresses in the substrate

$$\sigma_1 = -\frac{E_2(\alpha_1-\alpha_2)\Delta T}{\frac{1-\mu_2}{2}\frac{h_1}{h_2} + (1-\mu_1)\frac{E_2}{E_1}} \tag{7}$$

and in the coating

$$\sigma_2 = \frac{E_2(\alpha_1-\alpha_2)\Delta T}{1-\mu_2 + 2(1-\mu_1)\frac{E_2}{E_1}\frac{h_2}{h_1}} . \tag{8}$$

As the third example, we correct the initial stresses in a galvanic nickel coating (h_2=0.106mm; E_2=178 GPa, μ_2=0.31, α_2 =7.5×10^{-6} 1/°C) deposited bilaterally on a brass strip substrate (b=9.92 mm, h_1=0.15 mm; E_1=115 GPa, μ_1=0.34, α_1 =20.3×10^{-6} 1/°C) [3].

Fig. 5 presents the distribution of the initial stresses in the coating at a galvanizing temperature of 55°C and at a room temperature of 20°C. The initial stresses at galvanizing temperature are computed using expression [3]

$$\bar{\sigma} = 251\frac{0.106+h}{0.106+6.05h} .$$

The initial stresses at room temperature are obtained by adding the thermal stress σ_2 = -37.36 MPa calculated using formula (8).

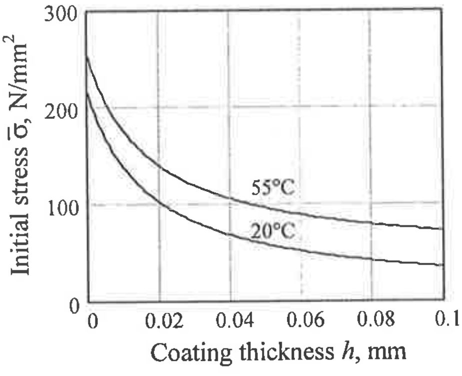

Fig. 5. Distribution of initial stresses in a galvanic nickel coating on a brass strip substrate at a galvanizing temperature of 55°C and at a room temperature of 20°C

4. Conclusions

1. Expressions are developed for thermal correction of residual stresses in coatings determined from the measured longitudinal deformations of a wire, tubular and strip substrates.
2. Three typical examples are presented for thermal correction of residual initial stresses in a galvanic nickel coating deposited on wire, tubular and strip substrates.

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