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DETERMINATION OF RESIDUAL STRESSES IN INHOMOGENEOUS PLATES WITH LAYER GROWING/REMOVING TECHNIQUES

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Abstract

A generalized algorithm of the layer growing/removing methods is presented for computing residual stresses in a free rectangular isotropic inhomogeneous plate whose elastic parameters depend on its thickness coordinate. The algorithm allows calculation of residual stresses from strains or curvatures measured on the stationary surface of the plate as well as from initial stresses measured on the moving surface using X-ray diffraction. The suggested algorithm is programmed for PC and presents interest first of all in the study of residual stresses in coatings and surface layers generally. An example of application is presented.

1. Nomenclature

E	Modulus of elasticity
h	Variable thickness of the plate
x, y, z	Rectangular coordinates
z_1	Initial thickness of the plate
z_2	Final thickness of the plate
μ_i	Poisson's ratio
σ_x, σ_y	Components of residual stress
$\overline{\sigma}_x, \overline{\sigma}_y$	Components of initial stress
σ^*_x, σ^*_y	Components of additional stress
$\varepsilon_x, \varepsilon_y$	Components of strain
$æ_x, æ_y$	Curvatures

2. Introduction

The layer removing method (destructive method) and the layer growing method (non-destructive method) are used for the determination of residual stresses in coated plates. The elaboration of the theory of the layer removing method started with papers by Treuting and Read [1] and by Moore and Evans [2] treating homogeneous plates. The theory of the layer growing method was evolved in works by Kõo [3-5], by Birger and Kozlov [6, 7], by Doi et al. [8, 9] as well as by other authors.

In a thesis [10] and in a paper [11] a common algorithm of the layer growing and layer removing methods is presented for the determination of equibiaxial residual stresses in an isotropic two-layer plate. This algorithm enables to calculate residual stresses from the strain or curvature measured on the stationary surface of the plate, as well as from initial stresses measured by the X-ray diffraction technique on the moving surface. In this study an advanced algorithm is presented that allows to calculate biaxial residual stresses in a free rectangular isotropic inhomogeneous elastic plate whose elastic parameters depend on its thickness coordinate continuously or piecewise.

3. A generalized algorithm of layer growing/removing methods for plates

Consider a thin layer growing on one face of a free rectangular plate (Fig. 1).

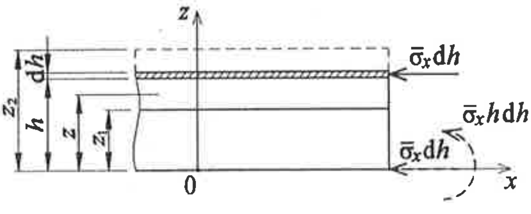

Figure 1. Layer-growing on the upper face of a rectangular plate

Let the initial thickness of the plate be z_1, variable thickness h and final thickness z_2. Rectangular coordinates x, y and z are used, where the free stationary surface of the plate is taken as the reference surface (x, y) and coordinate z is perpendicular to the stationary surface. It is assumed that axes x and y are principal axes of the state of residual stresses depending on the coordinate z only. It is also assumed that the edges of the plate are parallel to axes x and y.

The algorithm developed in this paper is based on the Kirchhoff small-deflection theory of plates [12] and on the general algorithm of the layer growing/removing methods [10, 11].

According to the mentioned general algorithm, residual stresses in a layer z of the coating can be calculated as a sum of initial and additional stresses:

$$\{\sigma\}=\{\bar{\sigma}\}+\{\sigma^*\} \tag{1}$$

where

$$\{\sigma\}=[\sigma_x \quad \sigma_y]^T,\ \{\bar{\sigma}\}=[\bar{\sigma}_x \quad \bar{\sigma}_y]^T,\ \{\sigma^*\}=[\sigma_x^* \quad \sigma_y^*]^T$$

are the vectors of residual stresses, initial stresses and additional stresses, respectively.

In order to express residual stresses from the measured deformation parameters we proceed from the mechanical effect of formation of the differential superficial layer dh (Fig. 1) at variable thickness h. As is known [10, 11], this effect can be expressed by applying differential edge forces to the edges of the differential superficial layer, which, when reduced to the reference surface, yield compressive edge forces $\bar{\sigma}_x \mathrm{d}h$, $\bar{\sigma}_y \mathrm{d}h$ and edge moments $\bar{\sigma}_x h \mathrm{d}h$, $\bar{\sigma}_y h \mathrm{d}h$.

Thus the problem is reduced to a problem for the free rectangular isotropic inhomogeneous plate subjected to compressive edge forces and bending edge moments along the edges.

In order to solve this problem we use, as was already noted, the Kirchhoff approximation. Initial stresses $\bar{\sigma}_x = \bar{\sigma}_x(h)$, $\bar{\sigma}_y = \bar{\sigma}_y(h)$ in the differential surface layer dh can be expressed by strains $\varepsilon_x = \varepsilon_x(h)$, $\varepsilon_y = \varepsilon_y(h)$, and curvatures $æ_x = æ_x(h)$, $æ_y = æ_y(h)$ measured on the stationary surface as follows:

$$\{\bar{\sigma}\} = [B]\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\} - [C]\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\} \tag{2}$$

$$h\{\bar{\sigma}\} = [C]\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\} - [D]\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\} \tag{3}$$

where

$$\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\} = \begin{bmatrix} \dfrac{\mathrm{d}\tilde{\varepsilon}_x}{\mathrm{d}h} & \dfrac{\mathrm{d}\tilde{\varepsilon}_y}{\mathrm{d}h} \end{bmatrix}^{\mathrm{T}}$$

is the vector of the derivatives of strain changes

$$\tilde{\varepsilon}_x = \varepsilon_x(z_2) - \varepsilon_x(h), \qquad \tilde{\varepsilon}_y = \varepsilon_y(z_2) - \varepsilon_y(h),$$

$$\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\} = \begin{bmatrix} \dfrac{\mathrm{d}\tilde{æ}_x}{\mathrm{d}h} & \dfrac{\mathrm{d}\tilde{æ}_y}{\mathrm{d}h} \end{bmatrix}^{\mathrm{T}}$$

is the vector of the derivatives of curvature changes $\tilde{æ}_x = æ_x(z_2) - æ_x(h)$, $\tilde{æ}_y = æ_y(z_2) - æ_y(h)$, and

$$[B] = \begin{bmatrix} B & B_\mu \\ B_\mu & B \end{bmatrix}, \quad [C] = \begin{bmatrix} C & C_\mu \\ C_\mu & C \end{bmatrix}, \quad [D] = \begin{bmatrix} D & D_\mu \\ D_\mu & D \end{bmatrix} \tag{4}$$

are the matrices of the elastic parameters given by

$$\begin{bmatrix} B & B_\mu \\ C & C_\mu \\ D & D_\mu \end{bmatrix} = \int_0^h [E^0(z)] \begin{Bmatrix} 1 \\ z \\ z^2 \end{Bmatrix} \mathrm{d}z \tag{5}$$

with

$$[E^0] = [E^0 \quad E^0_\mu] \tag{6}$$

and

$$E^0 = \frac{E}{1-\mu^2}, \qquad E^0_\mu = \mu E^0 \tag{7}$$

where $E = E(z)$ denotes the modulus of elasticity, and $\mu = \mu(z)$ denotes Poisson's ratio.

The expression for computing additional stresses in the coating $(z_1 \le z \le z_2)$ is

$$\{\sigma^*\} = [E^*]\int_z^{z_2} \left[-\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\} \quad z\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\} \right] \mathrm{d}h \tag{8}$$

where

$$[E^*] = E^0(z) \begin{bmatrix} 1 & \mu(z) \\ \mu(z) & 1 \end{bmatrix} \tag{9}$$

For computing residual stresses in the substrate $(0 \le z \le z_1)$ the lower limit z of the integral in expression (8) should be replaced by z_1.

If measurement of strains or curvatures is not performed during coating growth then, using removing procedure, it should be assumed in the above algorithm that

$$\varepsilon_x(z_2) = \varepsilon_y(z_2) = æ_x(z_2) = æ_y(z_2) = 0 .$$

Expressions (1)-(9) form a common algorithm of the layer growing/removing methods for isotropic inhomogeneous plates, allowing calculation of residual stresses at growing/removing on one face of the plate:

1. From strains and curvatures measured on the free stationary surface $(z = 0)$ depending on thickness h. In this case initial stresses are computed by using expression (2) or (3). From equation (8) the expression for computing additional stresses is
$$\{\sigma^*\} = [E^*][\{\tilde{\varepsilon}\} \quad - z\{\tilde{æ}\}] \tag{10}$$
2. From measured strains or curvatures only. In this case the unmeasured deformation parameter is computed from the equation
$$[[C] - h[B]]\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\} = [[D] - h[C]]\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\} \tag{11}$$
which follows from expressions (2) and (3).
3. From initial stresses measured by the X-ray diffraction technique on the moving surface depending on thickness h. In this case the derivatives of strain and curvature changes are calculated from expressions:
$$\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\} = \frac{[[D] - h[C]]}{[[B][D] - [C]^2]}\{\bar{\sigma}\} \tag{12}$$
$$\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\} = \frac{[[C] - h[B]]}{[[B][D] - [C]^2]}\{\bar{\sigma}\} \tag{13}$$
which are obtained by solving equations (2) and (3).

In special cases computation of residual stresses is simplified. For example, in case of the homogeneous plate (two-layered plate with equal elastic constants $\mu_1 = \mu_2 = \mu$, $E_1 = E_2 = E$) the Treuting-Read [1] and Moore-Evans [2] formulas follow from the above generalized algorithm.

Initial stresses are usually assumed to be equal in the determination of residual stresses in coatings. This hypothesis allows to obtain all special algorithms published earlier [10, 11 et al.].

4. Computer program RS-PLATE and computational example

On the basis of the presented algorithm (Sect. 3) a computer program RS-PLATE is written for PC in Turbo Pascal, which enables calculation of residual stresses in isotropic inhomogeneous plates from strains, curvatures or initial stresses measured during growing or removing process. According to the program, calculation of the derivatives of experimental data is carried out by a preliminary fitting with a polynomial.

As an example the program RS-PLATE is used for the determination of residual stresses with layer removing technique in the cold-rolled thermo-bimetallic plate, which characteristics are given in Table 1.

Table 1. Components and elastic constants of thermo-bimetallic specimen

Material	Components (%)	h (mm)	E (GPa)	$\alpha \cdot 10^6$ (1/°C)
Passive component	64Fe-36Ni	0.388	150	1.0
Active component	73Fe-24Ni-3Cr	0.372	170	18.5

Two strips 8.0x10.8 mm were cut from a sheet of 110x180x0.76 mm in lateral direction and two strips from the middle region of the sheet in longitudinal direction. After cutting the strips were cleaned and covered with shellac leaving 76 mm on one side uncovered for electropolishing.

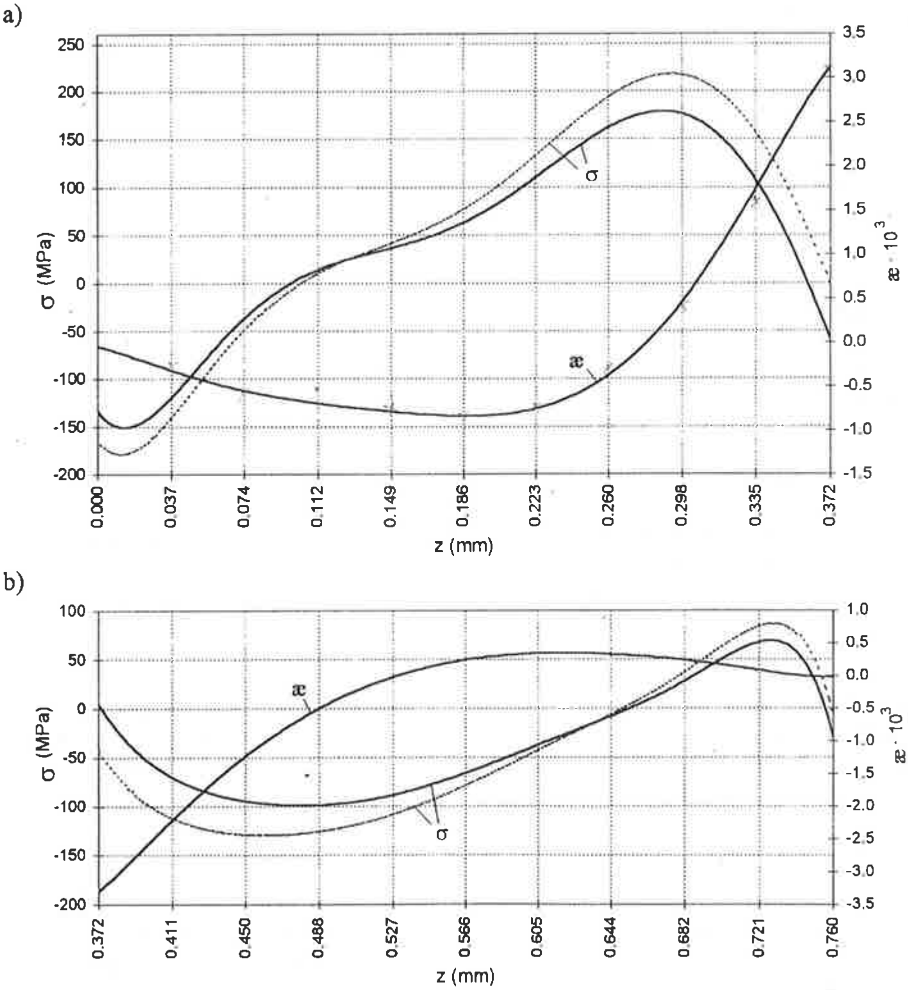

Figure 2. Experimental information (fitted) and residual stresses in a thermo-bimetallic plate in lateral direction at temperature 48 °C (solid lines) and 20 °C (dotted lines): a) in active component; b) in passive component

The active or passive components of the bimetal strips were removed in bath with liquid H_3PO_4 ~ 62%, H_2SO_4 ~ 19%, H_2O ~ 19% at current density 1.8 kA/m^2 and temperature 48 °C. The angular displacement of free end of the cantilevered strips were measured by a mirror instrument after each removed layer. By the results the changes of curvature depending on the thickness of removed layer were calculated, which were used as experimental information (Fig. 2). Calculated residual stresses $\sigma_x = \sigma_y = \sigma$ (Fig. 2) were reduced to temperature 20 °C by equations given in [13]. It can be seen that compressive and tensile stresses are present in both layers.

5. Conclusions

1. A generalized algorithm is elaborated for the computation of residual stresses in a free rectangular isotropic inhomogeneous along the thickness plate. The algorithm is universal and allows calculation of residual stresses at layer growing or layer removing from strains or curvatures measured on the stationary surface, or from initial stresses measured on the moving surface of the plate.
2. A computer program RS-PLATE based on the presented algorithm is introduced.
3. Using the program RS-PLATE residual stresses are computed in the cold-rolled thermo-bimetallic plate.

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