The Fifth International Conference on Residual Stresses

Volume 2

Edited by

T. Ericsson, M. Odén and A. Andersson

ICRS - 5

Layer Growing/Removing Method for the Determination of Residual Stresses in Inhomogeneous Cylinders

Jakub Kõo, Jaak Valgur
Estonian Agricultural University

Kreutzwaldi 5, EE2400 Tartu, Estonia
E-mail: koo@ph.eau.ee

Abstract

An algorithm of the layer growing/removing method is presented for computing residual stresses in case of the long multilayered hollow cylinder from strain measurements on the free stationary surface or from X-ray diffraction measurements on the cylinder's moving surface. The algorithm is valid for layer growing/removing on the outer or inner surface of the cylinder and is of interest first of all in the study of residual stresses in coatings and surface layers generally.

The suggested algorithm is programmed for PC. An example of application is presented.

1. Nomenclature

- E_i Modulus of elasticity of the i-th layer
- R Radius of the moving surface (superficial layer) of the cylinder
- r Radial coordinate
- r_0 Radius of the stationary free surface of the cylinder (substrate)
- r_i Radius of the interface of layers i and $i+1$
- r_n Radius of the free final surface of the coating
- r_k Radius of the interface of the substrate and coating
- k Number of substrate layers
- n Total number of substrate and coating layers
- μ_i Poisson's ratio of the i-th layer
- z Axial coordinate
- θ Angular coordinate
- $\sigma_{rm}, \sigma_{\theta m}, \sigma_{zm}$ Components of residual stress in the m-th layer
- $\overline{\sigma}_{\theta m}, \overline{\sigma}_{zm}$ Components of initial stress in the m-th layer
- $\sigma^*_{rm}, \sigma^*_{\theta m}, \sigma^*_{zm}$ Components of additional stress in the m-th layer
- $\varepsilon_{\theta(m)}, \varepsilon_{z(m)}$ Strains measured on the stationary surface of the cylinder at growing/removing of m-th layer

2. Introduction

The layer removing method (destructive method) and layer growing method (non-destructive method) [1, 2] are used for the determination of residual stresses in cylinders. The elaboration of the theory of the layer removing method started with papers [3–5] treating the homogeneous cylinder. In [6–10 et al.] the method is expanded to the inhomogeneous cylinder. The theory of the layer growing method is evolved in [11–13, 10 et al.]. In paper [14] a general algorithm of the layer growing and layer removing method is presented for the determination of residual stresses in layered cylinders. The algorithm enables to calculate residual stresses from circumferential and axial strains, measured on the inner surface of the hollow cylinder, depending on the radius of the moving outer surface. In the present paper a considerably developed algorithm is presented that allows to calculate residual stresses trough initial stresses measured by the X-ray diffraction method on the moving surface of the cylinder. The algorithm can be used in case of growing/removing either on the outer or inner surface of the cylinder.

3. General algorithm

Consider the thin layer growing on the outside of a multilayer cylinder (Fig. 1). Let the number of substrate layers be k, radius of the free surface r_0, radius of interface of the substrate and coating r_k. Layers are numerated starting from the layer adjacent to the free surface of the substrate towards the coating. Let the total number of substrate and coating layers be n, radius of the free surface of the coating r_n. Cylindrical coordinates r, θ, z are used, where coordinate z is taken along the axis of the cylinder.

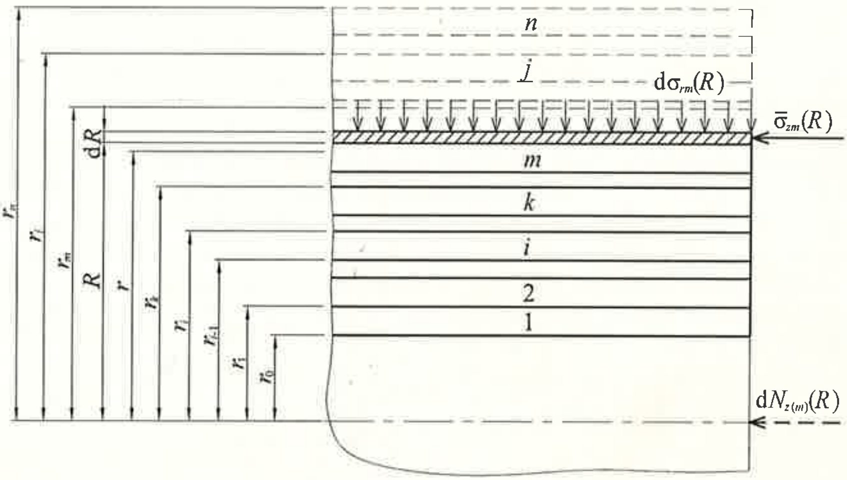

Figure 1. Layer-growing on the outside of a multilayered hollow cylinder

According to the general algorithm of the layer growing/removing method [1, 2], residual stresses in layer m of the coating can be calculated as a sum of initial and additional stresses:

$$\sigma_{rm}(r) = \sigma^*_{rm(m)}(r) + \sum_{j=m+1}^{n} \sigma^*_{rm(j)}(r)$$

$$\sigma_{\theta m}(r) = \overline{\sigma}_{\theta m}(r) + \sigma^*_{\theta m(m)}(r) + \sum_{j=m+1}^{n} \sigma^*_{\theta m(j)}(r) \tag{1}$$

$$\sigma_{zm}(r) = \overline{\sigma}_{zm}(r) + \sigma^*_{zm(m)}(r) + \sum_{j=m+1}^{n} \sigma^*_{zm(j)}(r)$$

where $\overline{\sigma}_{\theta m}(r)$, $\overline{\sigma}_{zm}(r)$ are initial circumferential and axial stresses in layer m, $\sigma^*_{rm(\;)}(r)$, $\sigma^*_{\theta m(\;)}(r)$, $\sigma^*_{zm(\;)}(r)$ are additional stresses in layer m, which arise during the growth of the same layer (index (m)) or any successive layer j (index (j)).

In order to express residual stresses from measured strain parameters we proceed from the mechanical effect of formation of the elementary superficial layer dR (Fig. 1) at variable radius R. As is known [1, 2], this effect can be expressed by applying the elementary surface load $\mathrm{d}\sigma_{rm}(R)$ and edge load $\overline{\sigma}_{zm}(R)\mathrm{d}R$. From equilibrium conditions of surface layer dR we obtain

$$\mathrm{d}\sigma_{rm}(R) = -\frac{\overline{\sigma}_{\theta m}(R)}{R}\mathrm{d}R \tag{2}$$

The edge load is replaced by axial force

$$\mathrm{d}N_{z(m)}(R) = 2\pi\,\overline{\sigma}_{zm}(R)R\mathrm{d}R \tag{3}$$

Thus the problem is reduced to an axially symmetric problem for the m-layered cylinder loaded with surface load (2) and axial force (3). In order to solve this problem we use the general solution of the axially symmetric problem of elasticity theory for the cylindrical layer [15]. Additional stresses arising in layer i due to the formation of layer m can be expressed as follows:

$$\left.\begin{matrix}\mathrm{d}\sigma^*_{ri(m)}(r,R)\\ \mathrm{d}\sigma^*_{\theta i(m)}(r,R)\end{matrix}\right\} = \frac{E_1^*}{2}\left[\mathrm{d}f_i(R) \mp \frac{r_0^2}{r^2}\mathrm{d}g_i(R)\right] \tag{4}$$

where $E_1^* = E_1/(1-\mu_1^2)$ and $f_i(R)$, $g_i(R)$ are dimensionless functions expressed by strains measured on the free surface of the substrate.

Solution (4) together with the equations proceeding from generalized Hooke's law and the hypothesis of plane cross-sections of the cylinder as well as equilibrium conditions for the part of the cylinder separated by the cross-section constitute general equations of the problem. The latter in combination with continuity conditions of radial stresses and circumferential strains on layers' interfaces as well as conditions for the free surface of the substrate ($r = r_0$) and moving surface of the coating ($r = R$) enable to express initial stresses and elementary additional stresses from strains measured on the free surface of the substrate. Passing over to radius r in equations of initial stresses and summarizing elementary additional stresses at the interval $r \le R \le r_m$ we obtain:

$$\begin{Bmatrix}\overline{\sigma}_{\theta m}(r)\\ \overline{\sigma}_{zm}(r)\end{Bmatrix} = \frac{E_1^*}{2}\begin{bmatrix}F_{1m}(r) & F_{2m}(r)\\ F_{3m}(r) & F_{4m}(r)\end{bmatrix}\begin{Bmatrix}\mathrm{d}\tilde{\varepsilon}_{\theta(m)}(r)/\mathrm{d}r\\ \mathrm{d}\tilde{\varepsilon}_{z(m)}(r)/\mathrm{d}r\end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix}\sigma^*_{rm(m)}(r)\\ \sigma^*_{\theta m(m)}(r)\\ \sigma^*_{zm(m)}(r)\end{Bmatrix} = \frac{E_1^*}{2}\begin{bmatrix}F_{5m}(r) & F_{6m}(r)\\ F_{7m}(r) & F_{8m}(r)\\ C_{1m} & C_{2m}\end{bmatrix}\begin{Bmatrix}\tilde{\varepsilon}_{\theta(m)}(r)\\ \tilde{\varepsilon}_{z(m)}(r)\end{Bmatrix} \tag{6}$$

where $\tilde{\varepsilon}_{\theta(m)}(r) = \varepsilon_{\theta(m)}(r_m) - \varepsilon_{\theta(m)}(r)$ and $\tilde{\varepsilon}_{z(m)}(r) = \varepsilon_{z(m)}(r_m) - \varepsilon_{z(m)}(r)$ are changes of circumferential and axial strains measured on the free surface of the substrate ($r = r_0$) during the growth of layer m at the interval $r \le R \le r_m$.

The elements of matrices in equations (5) and (6) are expressed:

$$F_{1m}(r) = r\left(A_{1m} - B_{1m}\frac{r_0^2}{r^2}\right), \qquad F_{2m}(r) = r\left(A_{2m} - B_{2m}\frac{r_0^2}{r^2}\right)$$

$$F_{3m}(r) = \frac{1}{r}\left[\sum_{i=1}^{m-1}\mu_i A_{1i}\left(r_i^2 - r_{i-1}^2\right) + \mu_m A_{1m}\left(r^2 - r_{m-1}^2\right)\right]$$

$$F_{4m}(r) = \frac{1}{r}\left[\sum_{i=1}^{m-1}\left(\frac{E_i}{E_1^*} + \mu_i A_{2i}\right)\left(r_i^2 - r_{i-1}^2\right) + \left(\frac{E_m}{E_1^*} + \mu_m A_{2m}\right)\left(r^2 - r_{m-1}^2\right)\right]$$

$$F_{5m}(r) = \frac{F_{1m}(r)}{r}, \qquad F_{6m}(r) = \frac{F_{2m}(r)}{r}, \qquad F_{7m}(r) = A_{1m} + B_{1m}\frac{r_0^2}{r^2}$$

$$F_{8m}(r) = A_{2m} + B_{2m}\frac{r_0^2}{r^2}, \qquad C_{1m} = 2\mu_m A_{1m}, \qquad C_{2m} = 2\left(\frac{E_m}{E_1^*} + \mu_m A_{2m}\right)$$

Constants A_{1m}, A_{2m}, B_{1m} and B_{2m} are calculated from recursion relations

$$A_{11} = B_{11} = 1, \qquad A_{21} = B_{21} = \mu_1$$

$$\left.\begin{aligned} A_{1i} &= a_{1i}A_{1,i-1} + a_{2i}B_{1,i-1} \\ A_{2i} &= a_{1i}A_{2,i-1} + a_{2i}B_{2,i-1} + a_{3i} \\ B_{1i} &= b_{1i}A_{1,i-1} + b_{2i}B_{1,i-1} \\ B_{2i} &= b_{1i}A_{2,i-1} + b_{2i}B_{2,i-1} + b_{3i} \end{aligned}\right\} \quad (i = 2, 3, \ldots, m)$$

In these formulas

$$a_{1i} = \frac{1}{2(1-\mu_i)}\left[1 + (1-2\mu_{i-1})\frac{1+\mu_{i-1}}{1+\mu_i}\frac{E_i}{E_{i-1}}\right]$$

$$a_{2i} = \frac{1}{2(1-\mu_i)}\frac{r_0^2}{r_{i-1}^2}\left(\frac{1+\mu_{i-1}}{1+\mu_i}\frac{E_i}{E_{i-1}} - 1\right), \qquad a_{3i} = (\mu_i - \mu_{i-1})\frac{E_i^*}{E_1^*}$$

$$b_{1i} = \frac{1}{2(1-\mu_i)}\frac{r_{i-1}^2}{r_0^2}\left[(1-2\mu_{i-1})\frac{1+\mu_{i-1}}{1+\mu_i}\frac{E_i}{E_{i-1}} - 1 + 2\mu_i\right]$$

$$b_{2i} = \frac{1}{2(1-\mu_i)}\left(1 - 2\mu_i + \frac{1+\mu_{i-1}}{1+\mu_i}\frac{E_i}{E_{i-1}}\right), \qquad b_{3i} = (\mu_i - \mu_{i-1})\frac{E_i^*}{E_1^*}\frac{r_{i-1}^2}{r_0^2}$$

where $E_i^* = E_i / (1-\mu_i^2)$.

Additional stresses in layer m, arising during the growth of the layer j $(j > m)$, are calculated according to relations (6):

$$\begin{Bmatrix} \sigma^*_{rm(j)}(r) \\ \sigma^*_{\theta m(j)}(r) \\ \sigma^*_{zm(j)}(r) \end{Bmatrix} = \frac{E_1^*}{2}\begin{bmatrix} F_{5m}(r) & F_{6m}(r) \\ F_{7m}(r) & F_{8m}(r) \\ C_{1m} & C_{2m} \end{bmatrix}\begin{Bmatrix} \tilde{\varepsilon}_{\theta(j)}(r_{j-1}) \\ \tilde{\varepsilon}_{z(j)}(r_{j-1}) \end{Bmatrix} \tag{7}$$

where $\tilde{\varepsilon}_{\theta(j)}(r_{j-1}) = \varepsilon_{\theta(j)}(r_j) - \varepsilon_{\theta(j)}(r_{j-1})$ and $\tilde{\varepsilon}_{z(j)}(r_{j-1}) = \varepsilon_{z(j)}(r_j) - \varepsilon_{z(j)}(r_{j-1})$ are the changes of circumferential and axial strains measured on the free surface of the substrate $(r = r_0)$ during the growth of the layer j $(r_{j-1} \le R \le r_j)$.

By solving equations (5) with respect to the derivatives of strains and by integrating in the limits $r \le R \le r_m$ we obtain formulas for calculating changes of strains from initial stresses measured on the moving surface of the coating:

$$\begin{Bmatrix} \tilde{\varepsilon}_{\theta(m)}(r) \\ \tilde{\varepsilon}_{z(m)}(r) \end{Bmatrix} = -\frac{2}{E_1^*}\int_r^{r_m}\frac{dR}{F_{1m}(R)F_{4m}(R) - F_{2m}(R)F_{3m}(R)}\begin{bmatrix} F_{4m}(R) & -F_{2m}(R) \\ -F_{3m}(R) & F_{1m}(R) \end{bmatrix}\begin{Bmatrix} \overline{\sigma}_{\theta m}(R) \\ \overline{\sigma}_{zm}(R) \end{Bmatrix} \tag{8}$$

Changes of strains $\tilde{\varepsilon}_{\theta(j)}(r_{j-1})$ and $\tilde{\varepsilon}_{z(j)}(r_{j-1})$ on the free surface of the substrate, arising from the growth of the layer j $(j > m)$, can be calculated by means of formulas (8) taking $r = r_{j-1}$ and $m = j$.

Equations (1), (5)-(8) form the general algorithm of the layer growing/removing methods for inhomogeneous cylinders, allowing of the calculation of residual stresses:

(1) at growing/removing on the outer or inner surface from strains measured on the free stationary surface of the cylinder. At growing/removing on the inner surface, the outer radius of the cylinder is r_0, and the layers are numerated from the outer layer of the substrate towards the centre of the cylinder. If the strains are not measured during the coating growth then $\varepsilon_{\theta(n)}(r_n) = \varepsilon_{z(n)}(r_n) = 0$;

(2) at growing/removing on the outer surface of the hollow or whole cylinder ($r_0 = 0$) from the initial stresses measured by the X-ray diffraction method on the moving outer surface;

(3) at growing/removing on the outer surface from axial strain (length change) assuming that initial circumferential and axial stresses are equal ($\overline{\sigma}_{\theta n}(r) = \overline{\sigma}_{\theta n}(r)$).

In order to use the given algorithm for a cylinder with elastic parameters changing continuously in the radial direction the cylinder is replaced by the multilayer cylinder. The values of the modulus of elasticity and Poisson's ratio on the middle surface of a layer are then taken as the values of these parameters for the layer.

In special cases the calculation of residual stresses is simplified. For example, in case of the cylinder with constant Poisson's ratio ($\mu_1 = \mu_i = \mu$) constants A_{2i}, B_{2i} are expressed by constants A_{1i}, B_{1i} as follows: $A_{2i} = \mu A_{1i}$, $B_{2i} = \mu B_{1i}$. For the homogeneous cylinder (two-layered cylinder with equal elastic parameters $\mu_1 = \mu_2 = \mu$, $E_1 = E_2 = E$) the Sachs-Espey [3, 4] and Moore-Evans [5] formulas follow from the expressions of the above general algorithm.

Initial circumferential and axial stresses are usually assumed to be equal in the determination of residual stresses in coatings. This hypothesis allows to obtain all earlier published special algorithms [1, 2] as special cases of the general algorithm.

4. Computer program RS-CYL and a computational example

On the basis of the presented algorithm (Sect. 3) the computer program RS-CYL is written for PC that enables to calculate residual stresses in multilayer cylinders from strain parameters or from initial stresses measured during growing or removing process.

In Figure 2 the distributions of residual stresses are presented in a hollow copper cylinder (r_0=5 mm, r_1=16 mm, E_1=110 GPa, μ_1=0.3) with a galvanic steel coating (r_2=21 mm, E_2=200 GPa, μ_2=0.3) calculated with the program RS-CYL. The stresses are calculated proceeding from the initial stresses $\overline{\sigma}_{\theta 2}(R) = \overline{\sigma}_{z2}(R) = \overline{\sigma}_2(R)$ determined from the strains measured during coating growth on the thin-walled tubular substrate at the coating temperature of +92.5°C [1, 2]. The stress distributions reduced to the temperature +20°C are calculated by means of the equations given in paper [1].

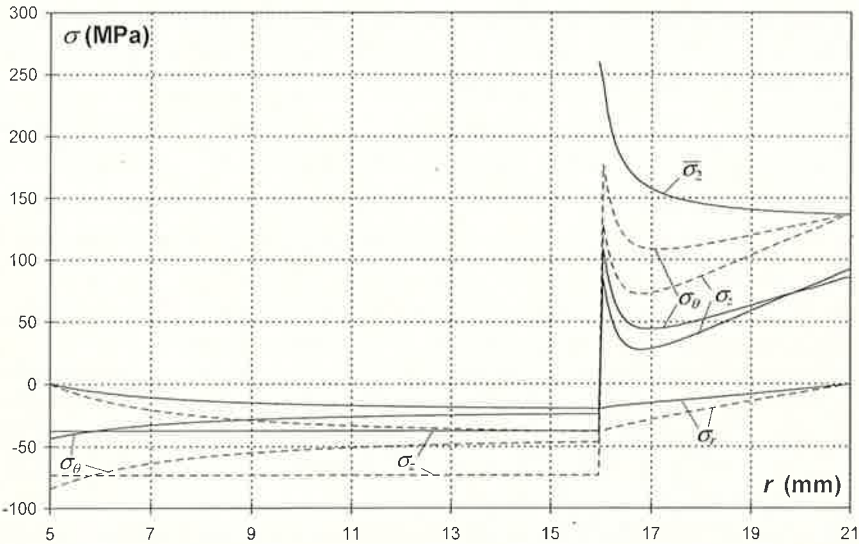

Figure 2. The distributions of initial and residual stresses in a hollow copper cylinder with a galvanic steel coating at temperatures +92.5°C (dashed lines) and +20°C (solid lines)

5. Conclusions

In this paper a general algorithm for the determination of residual stresses in the layered cylinders is elaborated. The algorithm is universal and allows to determine residual stresses at layer growing or layer removing from strain parameters measured on the free stationary surface or on the moving surface of the cylinder.

A computer program based on the present algorithm is introduced, and the results obtained by applying it to the coated hollow cylinder (layer growing on the external surface) is presented.

References

[1] Jakub Kõo. *Determination of Residual Stresses in Coatings and Coated Parts.* Dr. Eng. thesis, Series E. Machinery and Fine Mechanics, Tallinn Technical University, EE0026 Tallinn, Estonia, 1994.

[2] Jakub Kõo. *Layer-Growing and Layer-Removing Methods for Residual Stress Analysis: General Algorithm.* To appear in Proceedings of the 4th European Conference on Residual Stresses. Les Editions de Physique, 1997.

[3] G. Sachs. *Der Nachweis innerer Spannungen in Stangen und Rohren.* Zeitschrift für Metallkunde, 1927, **19**, 9, 352–357.

[4] G. Sachs, G. Espey. *The Measurement of Residual Stresses in Metal.* Iron Age, 1941, September 18, 63–71.

[5] M. G. Moore, W. P. Evans. *Mathematical Correction for Stress in Removed Layers in X-Ray Diffraction Residual Stress Analysis.* SAE Trans. 1958, **66**, 341–345.

[6] E. Krägeloh. *Bestimmung von Eigenspannungen an Zylindern aus inhomogenem Werkstoff mittels Ausbohr- und Abdrehverfahrens.* Materialprüf. 1959, **1**, 11/12, 377–384.

[7] Osamu Doi, Takayoshi Ukai, Atsumi Ohtsuki. *Measurement of Residual Stresses in Multi-Layer Electrodeposit Cylinders.* Trans. JSME, 1974, **40**, 329, 132–139 (in Japanese).

[8] Osamu Doi, Takayoshi Ukai, Atsumi Ohtsuki. *X-Ray Measurements of Residual Stresses in Multi-Layered Cylinder.* Bulletin of the JSME, 1975, **18**, 123, 940–952.

[9] U. Szieslo. *Berechnung von Eigenspannungen in metallgespritzten rotationssymmetrischen Körpern.* Z. Werkstofftech., 1980, **11**, 3, 110–115.

[10] D. A. Ignatkov. *Residual Stresses in Nonhomogeneous Parts.* Shtiintsa, Kishinev, 1992 (in Russian).

[11] J. P. Kõo. *Calculation of Residual Stresses in the Electrodeposited Circular Cylinders.* Zap. Leningr. Selskokhoz. in-ta, 1961, **82**, 179–190 (in Russian).

[12] L. I. Dekhtyar. *Determination of Residual Stresses in Coatings and Bimetals.* Kartya Moldovenyaskeh, Kishinev, 1968 (in Russian).

[13] Osamu Doi, Takayoshi Ukai, Atsumi Ohtsuki. *Measurements of Electrodeposition Stress in a Cylinder.* Trans. JSME, 1974, **40**, 331, 617-626 (in Japanese).

[14] J. Kõo. *Sur la détermination des contraintes résiduelles dans les cylindres creux non-homogènes.* In Th. Gaymann, editor, 6th International Conference on Experimental Stress Analysis, Preprints (VDI-Berichte, 1978, 313), VDI-Verlag GmbH, Düsseldorf, 1978, 487-492.

[15] S. P. Timoshenko, J. N. Goodier. *Theory of Elasticity.* McGraw-Hill, New-York, 1970.