Experimental Mechanics, Allison (ed.) © 1998 Balkema, Rotterdam, ISBN 90 5809 014 0

Residual stress determination in plates using layer growing/removing methods

J. Kõo & J. Valgur
Chair of Structural Mechanics and Engineering, Estonian Agricultural University, Tartu, Estonia

ABSTRACT: A generalized algorithm of the layer growing/removing methods is presented for computing residual stresses in a free rectangular orthotropic inhomogeneous plate whose elastic parameters depend on its thickness coordinate. The algorithm allows calculation of residual stresses from strains or curvatures measured on the stationary surface of the plate as well as from initial stresses measured on the moving surface using X-ray diffraction. The suggested algorithm is programmed for PC and presents interest first of all in the study of residual stresses in coatings and surface layers generally. Three examples of application are presented.

1. INTRODUCTION

The layer removing method (destructive method) and the layer growing method (non-destructive method) are used for the determination of residual stresses in coated plates. The elaboration of the theory of the layer removing method started with papers by Treuting and Read (1951) and by Moore and Evans (1958) treating homogeneous plates. The theory of the layer growing method was evolved in works by Kõo (1959, 1969, 1979), by Birger and Kozlov (1974), by Doi et al. (1974, 1975) as well as by other authors.

In a thesis Kõo (1994) and in a paper by same author (1997) a common algorithm of the layer growing and layer removing methods is presented for the determination of equibiaxial residual stresses in an isotropic two-layer plate. This algorithm enables to calculate residual stresses from the strain or curvature measured on the stationary surface of the plate, as well as from initial stresses measured by the X-ray diffraction technique on the moving surface. In this study an advanced algorithm is presented that allows to calculate biaxial residual stresses in a free rectangular orthotropic inhomogeneous elastic plate whose elastic parameters depend on its thickness coordinate continuously or piecewise.

2. A GENERALIZED ALGORITHM OF LAYER GROWING/REMOVING METHODS FOR PLATES

Consider a thin layer growing on one face of a free rectangular plate (Fig. 1). Let the initial thickness of the plate be z_1, variable thickness h and final thickness z_2. Rectangular coordinates x, y and z are used, where the free stationary surface of the plate is taken as the reference surface (x, y) and coordinate z is perpendicular to the stationary surface. It is assumed that axes x and y are both orthotropic axes and principal axes of the state of residual stresses depending on the coordinate z only. It is also assumed that the edges of the plate are parallel to axes x and y.

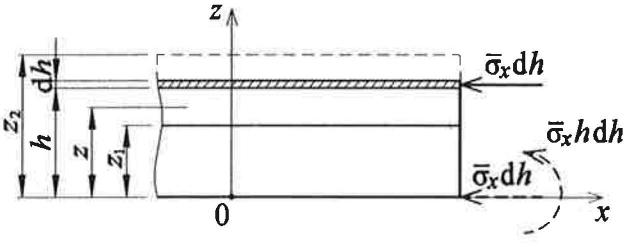

Figure 1. Layer-growing on the upper face of a rectangular plate.

The algorithm developed in this paper is based on the Kirchhoff small-deflection theory of plates (Timoshenko & Woinowski-Krieger 1959) and on the general algorithm of the layer growing/removing methods (Kõo 1994, 1997).

According to the mentioned general algorithm, residual stresses in a layer z of the coating can be calculated as a sum of initial and additional stresses:

$$\{\sigma\}=\{\bar{\sigma}\}+\{\sigma^*\} \tag{1}$$

where

$$\{\sigma\}=\begin{bmatrix}\sigma_x & \sigma_y\end{bmatrix}^{\mathrm{T}}$$

$$\{\bar{\sigma}\}=\begin{bmatrix}\bar{\sigma}_x & \bar{\sigma}_y\end{bmatrix}^{\mathrm{T}}$$

$$\{\sigma^*\}=\begin{bmatrix}\sigma_x^* & \sigma_y^*\end{bmatrix}^{\mathrm{T}}$$

are the vectors of residual stresses, initial stresses and additional stresses, respectively.

In order to express residual stresses from the measured deformation parameters we proceed from the mechanical effect of formation of the differential superficial layer dh (Fig. 1) at variable thickness h. As is known (Kõo 1994, 1997), this effect can be expressed by applying differential edge forces to the edges of the differential superficial layer, which, when reduced to the reference surface, yield compressive edge forces $\bar{\sigma}_x dh$, $\bar{\sigma}_y dh$ and edge moments $\bar{\sigma}_x h dh$, $\bar{\sigma}_y h dh$.

Thus the problem is reduced to a problem for the free rectangular orthotropic inhomogeneous plate subjected to compressive edge forces and bending edge moments along the edges.

In order to solve the problem we use, as was already noted, the Kirchhoff approximation. Initial stresses $\bar{\sigma}_x=\bar{\sigma}_x(h)$, $\bar{\sigma}_y=\bar{\sigma}_y(h)$ in the differential surface layer dh can be expressed by strains $\varepsilon_x=\varepsilon_x(h)$, $\varepsilon_y=\varepsilon_y(h)$, and curvatures $æ_x=æ_x(h)$, $æ_y=æ_y(h)$ measured on the stationary surface as follows:

$$\{\bar{\sigma}\}=[B]\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\}-[C]\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\} \tag{2}$$

$$h\{\bar{\sigma}\}=[C]\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\}-[D]\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\} \tag{3}$$

where

$$\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\}=\begin{bmatrix}\frac{\mathrm{d}\tilde{\varepsilon}_x}{\mathrm{d}h} & \frac{\mathrm{d}\tilde{\varepsilon}_y}{\mathrm{d}h}\end{bmatrix}^{\mathrm{T}}$$

is the vector of the derivatives of strain changes

$\tilde{\varepsilon}_x=\varepsilon_x(z_2)-\varepsilon_x(h)$, $\tilde{\varepsilon}_y=\varepsilon_y(z_2)-\varepsilon_y(h)$,

$$\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\}=\begin{bmatrix}\frac{\mathrm{d}\tilde{æ}_x}{\mathrm{d}h} & \frac{\mathrm{d}\tilde{æ}_y}{\mathrm{d}h}\end{bmatrix}^{\mathrm{T}}$$

is the vector of the derivatives of curvature changes $\tilde{æ}_x=æ_x(z_2)-æ_x(h)$, $\tilde{æ}_y=æ_y(z_2)-æ_y(h)$, and

$$[B]=\begin{bmatrix}B_x & B_\mu\\ B_\mu & B_y\end{bmatrix} \tag{4}$$

$$[C]=\begin{bmatrix}C_x & C_\mu\\ C_\mu & C_y\end{bmatrix} \tag{5}$$

$$[D]=\begin{bmatrix}D_x & D_\mu\\ D_\mu & D_y\end{bmatrix} \tag{6}$$

are the matrices of the elastic parameters given by

$$\begin{bmatrix}B_x & B_y & B_\mu\\ C_x & C_y & C_\mu\\ D_x & D_y & D_\mu\end{bmatrix}=\int_0^h\left[E^0(z)\right]\begin{Bmatrix}1\\ z\\ z^2\end{Bmatrix}\mathrm{d}z \tag{7}$$

with

$$\left[E^0\right]=\begin{bmatrix}E_x^0 & E_y^0 & E_\mu^0\end{bmatrix} \tag{8}$$

and

$$E_x^0=\frac{E_x}{1-\mu_{xy}\mu_{yx}},\quad E_y^0=\frac{E_y}{1-\mu_{xy}\mu_{yx}} \tag{9}$$

$$E_\mu^0=\frac{\mu_{xy}E_x}{1-\mu_{xy}\mu_{yx}}=\frac{\mu_{yx}E_y}{1-\mu_{xy}\mu_{yx}} \tag{10}$$

where $E_x=E_x(z)$, $E_y=E_y(z)$ denote the orthotropic moduli of elasticity, and $\mu_{xy}=\mu_{xy}(z)$, $\mu_{yx}=\mu_{yx}(z)$ denote orthotropic Poisson's ratios.

The expression for computing additional stresses in the coating $(z_1\le z\le z_2)$ is

$$\{\sigma^*\}=\left[E^*\right]\int_z^{z_2}\left[-\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\}\quad z\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\}\right]\mathrm{d}h \tag{11}$$

where

$$\left[E^*\right]=\begin{bmatrix}E_x^0(z) & E_\mu^0(z)\\ E_\mu^0(z) & E_y^0(z)\end{bmatrix} \tag{12}$$

For computing residual stresses in the substrate $(0\le z\le z_1)$ the lower limit z of the integral in expression (11) should be replaced by z_1.

If measurement of strains or curvatures is not performed during coating growth then, using removing procedure, it should be assumed in the above algorithm that

$\varepsilon_x(z_2)=\varepsilon_y(z_2)=æ_x(z_2)=æ_y(z_2)=0$.

Expressions (1)-(12) form a common algorithm of the layer growing/removing methods for orthotropic inhomogeneous plates, allowing calculation of residual stresses at growing/removing on one face of the plate:

1. From strains and curvatures measured on the free stationary surface ($z=0$) depending on thickness h. In this case initial stresses are computed by using expression (2) or (3). From equation (11) the expression for computing additional stresses is

$$\{\sigma^*\}=[E^*][\{\tilde{\varepsilon}\} \; -z\{\tilde{æ}\}] \tag{13}$$

2. From measured strains or curvatures only. In this case the unmeasured deformation parameter is computed from the equation

$$[[C]-h[B]]\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\}=[[D]-h[C]]\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\} \tag{14}$$

which follows from expressions (2) and (3).

3. From initial stresses measured by the X-ray diffraction technique on the moving surface depending on thickness h. In this case the derivatives of strain and curvature changes are calculated from expressions

$$\left\{\frac{\mathrm{d}\tilde{\varepsilon}}{\mathrm{d}h}\right\}=\frac{[[D]-h[C]]}{[[B][D]-[C]^2]}\{\bar{\sigma}\} \tag{15}$$

$$\left\{\frac{\mathrm{d}\tilde{æ}}{\mathrm{d}h}\right\}=\frac{[[C]-h[B]]}{[[B][D]-[C]^2]}\{\bar{\sigma}\} \tag{16}$$

which are obtained by solving equations (2) and (3).

In special cases computation of residual stresses is simplified. For example, in case of the homogeneous plate (two-layered plate with equal elastic constants $\mu_1=\mu_2=\mu$, $E_1=E_2=E$) the Treuting-Read (1951) and Moore-Evans (1958) formulas follow from the above generalized algorithm.

Initial stresses are usually assumed to be equal in the determination of residual stresses in coatings. This hypothesis allows to obtain all special algorithms published earlier (Kõo 1994 et al.) as special cases of the generalized algorithm.

3. COMPUTER PROGRAM RS-PLATE AND COMPUTATIONAL EXAMPLES

On the basis of the presented algorithm (Sect. 2) a computer program RS-PLATE is written for PC in Turbo Pascal, which enables calculation of residual stresses in orthotropic inhomogeneous plates from strains, curvatures or initial stresses measured during growing or removing process. According to the program, calculation of the derivatives of experimental data is carried out by a preliminary fitting with a polynomial.

Using the program RS-PLATE three computational examples are realized.

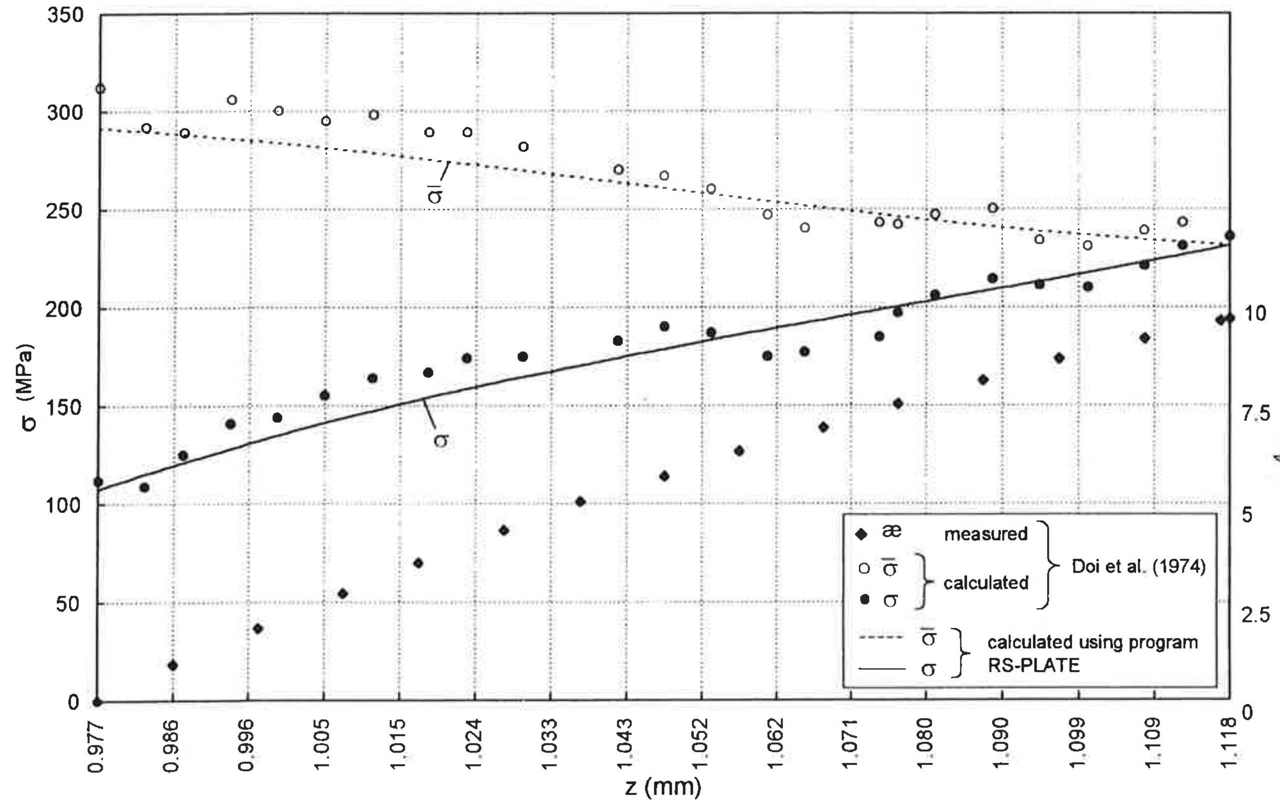

Figure 2. Experimental information and distribution of initial and residual stresses in a galvanic nickel coating.

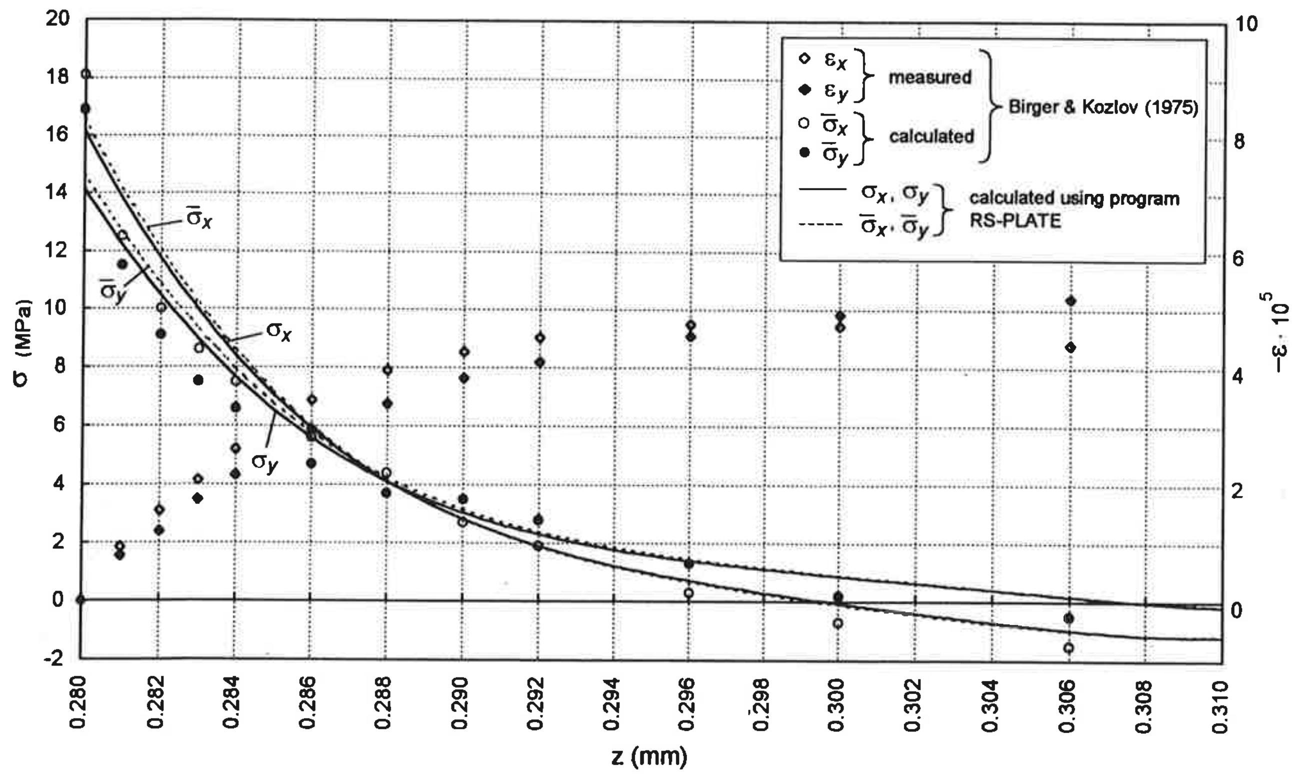

Figure 3. Experimental information and distribution of initial and residual stresses in a polycristallic tellurium layer.

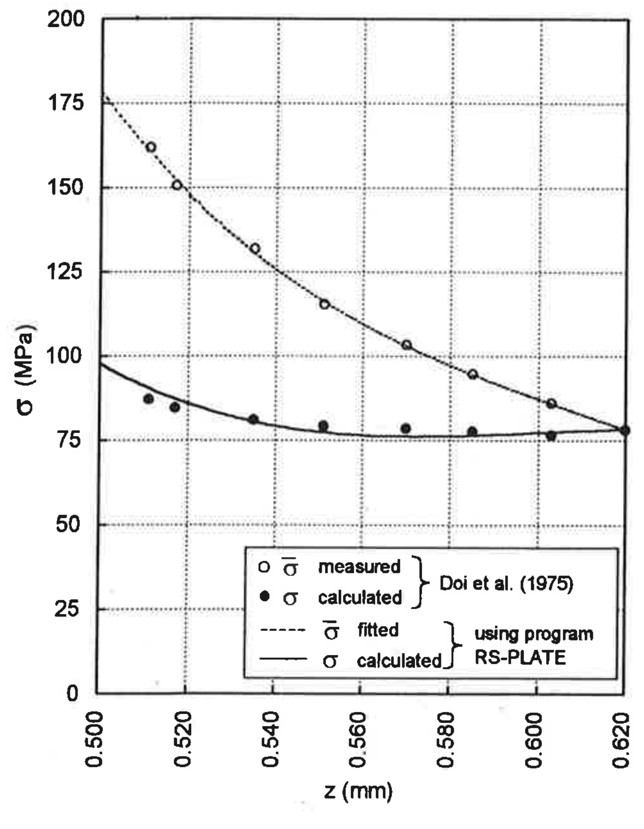

Figure 4. Experimental information and distribution of initial and residual stresses in a galvanic chromium coating.

As the first example, equibiaxial residual stresses are computed in a galvanic nickel coating from curvatures measured by Doi et al. (1974) during the growing process of the coating on a copper plate substrate. Figure 2 shows the experimental data of curvature measurements ($æ_x = æ_y = æ$) and the distribution of initial and residual stresses ($\bar{\sigma}_x = \bar{\sigma}_y = \bar{\sigma}$, $\sigma_x = \sigma_y = \sigma$) in the coating ($z_2 = 1.118\,\text{mm}$, $\mu_2 = 0.31$, $E_2 = 207\,\text{GPa}$) deposited on a plate substrate ($z_1 = 0.977\,\text{mm}$, $\mu_1 = 0.34$, $E_1 = 122.6\,\text{GPa}$). Considerable scattering of the results of computation obtained by Doi et al. (1974) can be evidently explained by the omission of experimental data fitting.

As the second example, residual stresses are computed in the surface layer of an isotropic bimetallic (monocristallic germanium + polycristallic tellurium) plate from strains measured by Birger and Kozlov (1975) during the removing process of a tellurium layer. Figure 3 presents the experimental data of strain measurements and the distribution of initial and residual stresses in the tellurium layer ($z_2 = 0.31\,\text{mm}$, $\mu_2 = 0.188$, $E_2 = 38.95\,\text{GPa}$) on a germanium plate ($z_1 = 0.28\,\text{mm}$, $\mu_1 = 0.274$, $E_1 = 102.02\,\text{GPa}$). As one can see, initial stresses by Birger and Kozlov (1975) differ slightly from those computed using program RS-PLATE. This is due, among other things, to the averaging of the Poisson's ratios of germanium and tellurium in the calculations of Birger and Kozlov. It is also evident that initial and residual stresses differ slightly, i.e. additional stresses in thin coatings are negligible (see Kõo 1994, 1997).

As the third example, we present the results of computing equibiaxial residual stresses in a galvanic chromium coating from initial stresses measured by Doi et al. (1975) during the process of removing of the coating from a steel plate substrate using X-ray technique. Figure 4 shows the experimental data of initial stress measurements ($\bar{\sigma}_x = \bar{\sigma}_y = \bar{\sigma}$) and the distribution of initial and residual stresses ($\sigma_x = \sigma_y = \sigma$) in the coating ($z_2 = 0.62\,\text{mm}$, $\mu_2 = 0.23$, $E_2 = 181.5\,\text{GPa}$) deposited on a steel plate substrate ($z_1 = 0.5\,\text{mm}$, $\mu_1 = 0.28$, $E_1 = 192.3\,\text{GPa}$). It can be seen that residual stresses determined by Doi et al. (1975) agree satisfactorily with those computed using program RS-PLATE.

4. CONCLUSIONS

1. A generalized algorithm is elaborated for the computation of residual stresses in a free orthotropic inhomogeneous plate. The algorithm is universal and allows calculation of residual stresses at layer growing or layer removing from strains or curvatures measured on the stationary surface, or from initial stresses measured on the moving surface of the plate.
2. A computer program RS-PLATE based on the presented algorithm is introduced.
3. Using the program RS-PLATE residual stresses are computed in the galvanic nickel coating on a copper plate, in the tellurium layer on a germanium plate and in the galvanic chromium coating on a steel plate.

REFERENCES

Birger, I. A. & M. L. Kozlov 1974. Determination of residual stresses in thin coatings of orthotropic plates. (In Russian). *Zavodskaya Laboratoriya* 40 (2): 223-225.

Birger, I. A. & M. L. Kozlov 1975. Determination of residual stresses in a plate whose elastic parameters are variable through thickness. (In Russian). *Zavodskaya Laboratoriya* 41 (2): 239-241.

Doi, O., T. Ukai & A. Ohtsuki 1974. Measurements of Electrodeposition Stress in a Plate Cathode. *Bull. Jap. Mech. Eng.* 17 (105): 297-304.

Doi, O. & T. Ukai 1975. Residual Stress Measurement of Multi-Layered Plate by X-Ray Method. *Bull. Jap. Mech. Eng.* 18 (119): 493-500.

Kõo, J. 1959. Calculation of residual stresses in the electrodeposits from deformation of a thin plate cathode. (In Russian). *Trans. Est. Agricult. Acad.* 13: 63-75.

Kõo, J. P. 1969. Determination of residual stresses in coatings of bars and plates using strain measurement method. (In Russian). *Izvestiya vuzov.* Mashinostroenie 12: 47-51.

Kõo, J. P. 1979. Determination of residual stresses in coatings of plates using curvature measurement method. (In Russian). *Izvestiya vuzov.* Mashinostroenie 8: 9-12.

Kõo, J. 1994. Determination of Residual Stresses in Coatings and Coated Parts. *Dr. Eng. Thesis, Series E. Machinery and Fine Mechanics.* Tallinn Technical University, Estonia.

Kõo, J. 1997. Layer-Growing and Layer-Removing Methods for Residual Stress Analysis. Accepted for publication in *Proc. of the 4th Europ. Conf. on Residual Stresses, Cluny (France), 4-6 June 1996:* 10 p.

Moore, M. G. & W. P. Evans 1958. Mathematical Correction for Stress in Removed Layers in X-Ray Diffraction Residual Stress Analysis. *SAE Trans.* 66: 340-345.

Perakh, M. 1979. Calculation of spontaneous macrostresses in deposits from deformation of substrates and from restoring force factors. *Surf. Technol.* 8: 265-309.

Timoshenko, S. & S. Woinowsky-Krieger 1959. *Theory of Plates and Shells.* New York: McGraw-Hill.

Treuting, R. G. & W. T. Read 1951. A Mechanical Determination of Biaxial Residual Stress in Sheet Materials. *J. Appl. Phys.* 22 (2): 130-134.